MACHINE DESIGN

NOTES

ON

MACHINE DESIGN

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Preface to Second Edition.

In presenting this book no claim is made of originality of subject matter, as nearly every thing in it can be found elsewhere. The object in preparing the book was to gather together in small compass the more simple formulas for the strength and stiffness of machine parts, with an explanation of the principles involved, and with such tables and general information as the designer of machinery might find useful.

The book pre-supposes an acquaintance with mathematics and the laws of the strength of materials.

In short, the aim has been to put the mathematical principles of macnine design in a compact form at a moderate price for the use of the student and the young engineer.

In revising the text for a second edition some additions have been made to the physical constants in tables I and II as the result of recent experiments. Experimental data obtained by the author in the laboratories of the school have also been added, notably those in regard to iron and steel pulleys, belts, fly wheels, gear teeth, ball bearings, and the friction of steam packings.

The author wishes to acknowledge the great assistance given him by Mr. J. Verne Stanford in the preparation of drawings for the cuts in this edition.

Chapter I.

UNITS AND TABLES.

1. Units. In this book the following units will be used unless otherwise stated.

Dimensions in inches.
Forces in pounds.
Stresses in pounds per square inch.
Velocities in feet per second.
Work and energy in foot pounds.
Moments in pounds inches.
Speeds of rotation in revolutions per minute.

The word *stress* will be used to denote the resistance of material to distortion per unit of sectional area. The word *strain* will be used to denote the distortion of a piece per unit of length. The word *set* will be used to denote total permament distortion of a piece.

In making calculations the use of the slide-rule and of four-place logarithms is recommended; accuracy is expected only to three significant figures.

2. Materials. The principal materials used in machine construction are given in the following tables with the physical characteristics of each.

By *wrougt iron* is meant commercially pure iron which has been made from molten pig-iron by the puddling process and then squeezed and rolled, thus developing the fiber. This iron has been largely supplanted by soft steel.

In making *steel*, on the other hand, the molten iron has had the silicon and carbon removed by a hot blast, either passing through the liquid as in the Bessemer converter, or over its surface as in the open-hearth furnace. A suitable quantity of carbon and manganese has then been added and the metal poured into ingot molds. If the steel is then reheated and passed through

a series of rolls, structural steel and rods or rails result.

Steel castings are poured directly from the open hearth furnace and allowed to cool without any drawing or rolling. They are coarser and more crystalline than the rolled steel.

Open hearth steel is generally used for boiler plates and of these, two grades are commonly known as marine steel and flange steel.

Bessemer steel is largely used in the manufacture of rails for steam and electric roads.

Crucible steel usually contains from one to one and a half per cent of carbon, is relatively high priced and only used for cutting tools. It is made by melting steel in an air tight crucible with the proper additions of carbon and manganese.

Cast iron is made directly from the pig by remelting and casting, is granular in texture and contains from two to five per cent. of carbon. A portion of the carbon is chemically combined with the iron while the remainder exists in the form of graphite. The harder and whiter the iron the more carbon is found chemically combined. Silicon is an important element in cast iron and influences the rate of cooling. The more slowly iron cools after melting the more graphite forms and the softer the iron.

Malleable iron is cast iron annealed and partially decarbonized by being heated in an annealing oven in contact with some oxidising material such as hæmatite ore. This process makes the iron tougher and less brittle

All castings including those made from alloys are somewhat unreliable on account of hidden flaws and of the strains developed by shrinkage while cooling.

The constants for strength and elasticity are only fair average values, and should be determined for any special material by direct experiment when it is practicable. Many of the constants are not given in the table on account of the lack of reliable data for their determination.

TABLE I.—WROUGHT METALS.

Kind of Metal.	Wt. of Cu. Inch	Wt. of Cu. Ft.	Ultimate Strength. Tensi'n	Ultimate Strength. Compress	Ultimate Strength. Shear	Elastic Limit. Tension.	Modulus of Rupture Tr'nsv'rse	Modulus of Elasticity. Tension.
Wrought Iron, small bars......	.28	485	55000	38000	45000	28000	40000	26000000
Wrought Iron, plates............	...	...	50000	......	40000	25000	......	25000000
Wrought Iron, large forgings	...	...	45000	.....	35000	22500	30000	25000000
Steel, flange plate..............	...	...	58000	100000	48000	34000	......	28000000
Steel, marine plate.......	...	...	52000	......	......	30000	......	24000000
Soft Steel, 0.15 C.............	...	...	65000	......	50000	35000	......	28000000
Machinery Steel, 0.55 C......	...	...	80000	......	65000	45000	......	30000000
Steel, Crucible or Tool...... ..	.282	487	120000	......	......	60000	.....	40000000

Prof. Thurston gives the following formula for the tensile strength of steel when C is the per cent of carbon: S + 60000=70000C.

TABLE II.—CAST METALS.

Kind of Metal.	Wt. of Cu. Inch	Wt. of Cu. Ft.	Ultimate Strength.			Elastic Limit.	Modulus of Rupture	Modulus of Elasticity.
			Tensi'n	Compress	Shear	Tension.	Transv'rse	Tension.
Cast Iron	.26	450	18000	75000	25000	12000	36000	18000000
Malleable Castings	.256	442	36000	.. ..	42000	16000	....	....
Steel Castings	.....	......	38000	125000	....	18000	....	30000000
Brass Castings.........	.289	500	18000	12000	....	. ...	....	9000000
Copper Castings...............	.321	555	24000	75000	24000	....	30000	15000000
Bronze, Gun Metal............	.309	534	36000	100000	. ...	....	....	10000000
Bronze, 10Al. 90 Cu.	......	.....	85000	132000	....	....	....	....
Bronze. Phosphor.............	.....	......	58000	....	43000	20000	....	14000000
Aluminum Castings........ ...	.092	159	28000	13000	....	14000	....	11000000
Aluminum Wire...............	.....	.....	42000	....	....	....	....	....

3. Notation.

Let S = Stress per square inch.
W = Total load applied in pounds.
M = Bending moment in pounds inches.
T = Twisting moment in pounds inches.
b = Breadth of cross-section in inches.
h = Depth of cross-section in inches.
d = Diameter of circular section in inches.
A = Area of cross-section in square inches.
l = Length of piece in inches.
I = Rectangular moment of inertia.
J = Polar moment of inertia.
c = Half depth of beam or shaft in inches.
r = Radius of gyration of section in inches.
$\frac{I}{c}$ = Section modulus for bending.
$\frac{J}{c}$ = Section modulus for twisting.

4. Formulas.

Simple Stress.

Tension, compression or shear, $S=\frac{W}{A}$ .... (1)

Bending under Transverse Load.

General equation, $M=\frac{SI}{c}$ .... (2)

Rectangular section, $M=\frac{Sbh^2}{6}$ .... (3)

Rectangular section, $bh^2=\frac{6M}{S}$ .... (4)

Circular section, $M=\frac{Sd^3}{10.2}$ .... (5)

Circular section, $d=\sqrt[3]{\frac{10.2M}{S}}$ .... (6)

Torsion or Twisting.

General equation, $T=\frac{SJ}{c}$ .... (7)

Circular section, $$T = \frac{Sd^3}{5.1} \quad \ldots\ldots\ldots\ldots (8)$$

Circular section, $$d = \sqrt[3]{\frac{5.1\ T}{S}} \quad \ldots\ldots\ldots (9)$$

Hollow circular section, $$T = \frac{S}{5.1} \frac{d^4 - d_1^4}{d} \quad \ldots\ldots (10)$$

Other values of $\frac{I}{c}$ and $\frac{J}{c}$ may be taken from Table 4.

Combined Bending and Twisting.

Calculate shaft for a twisting moment,

$$T^1 = M + \sqrt{M^2 + T^2} \quad \ldots\ldots\ldots\ldots\ldots (11)$$

Column subject to Bending.

Use Rankine's formula, $$\frac{W}{A} = \frac{S}{1 + q\frac{l^2}{r^2}} \quad \ldots\ldots\ldots\ldots\ldots (12)$$

The values of r^2 may be taken from Table IV. The subjoined table gives the average values of q, while S is the compressive strength of the material.

TABLE III.—Values of q in formula 12.

Material.	Both ends fixed.	Fixed and round.	Both ends round.	Fixed and free.
Timber...........	$\frac{1}{3000}$	$\frac{1.78}{3000}$	$\frac{4}{3000}$	$\frac{16}{3000}$
Cast Iron	$\frac{1}{5000}$	$\frac{1.78}{5000}$	$\frac{4}{5000}$	$\frac{16}{5000}$
Wrought Iron...	$\frac{1}{36000}$	$\frac{1.78}{36000}$	$\frac{4}{36000}$	$\frac{16}{36000}$
Steel........	$\frac{1}{25000}$	$\frac{1.78}{25000}$	$\frac{4}{25000}$	$\frac{16}{25000}$

In this formula, as in all such, the values of the constants should be determined for the material used by direct experiment if possible.

Or use straight line formula, $\frac{W}{A} = S - k\frac{l}{r}$........(12a)

TABLE IIIa.—Values of S and k in formula (12a).
(Merriman's Mechanics of Materials.)

Kind of Column.	S	k	Limit $\frac{l}{r}$
Wrought Iron :			
Flat ends	42000	128	218
Hinged ends	42000	157	178
Round ends	42000	203	138
Mild Steel :			
Flat ends..................	52500	179	195
Hinged ends.............	52500	220	159
Round ends	52500	284	123
Cast Iron :			
Flat ends..................	80000	438	122
Hinged ends..............	80000	537	99
Round ends	80000	693	77
Oah :			
Flat ends..................	5400	28	128

See also Carnegie's Pocket Companion (pp. 129, 147 and 152) for applications of these formulas.

For values of $\frac{l}{r}$ less than 90 mild steel columns are calculated for direct compression.

TABLE IV.—CONSTANTS OF CROSS-SECTION.

Form of Section and Area A	Square of Radius of Gyration. r^2	Moment of Inertia $I=Ar^2$	Section Mod'lus $\frac{I}{c}$	Polar Moment of Inertia. J	Tortion Mod'lus $\frac{J}{c}$
Rect'ngle bh	$\frac{h^2}{12}$	$\frac{bh^2}{12}$	$\frac{bh^2}{6}$	$\frac{bh^3+bh^3}{12}$	$\frac{bh^3+b^3h}{6\sqrt{b_j^2+h^2}}$
Square d^2	$\frac{d^2}{12}$	$\frac{d^4}{12}$	$\frac{d^3}{6}$	$\frac{d^4}{6}$	$\frac{d^3}{4.24}$
Hollow Rect'ngle or I-beam $bh—b_1h_1$	$\frac{bh^3-b_1h_1^3}{(12bh-b_1h_1)}$	$\frac{bh^3-b_1h_1^3}{12}$	$\frac{bh^3-b^1h_1^3}{6h}$		
Circle $\frac{\pi}{4}d_2$	$\frac{d^2}{16}$	$\frac{\pi d^4}{64}$	$\frac{d^3}{10.2}$	$\frac{\pi d^4}{32}$	$\frac{d^3}{5.1}$
Hollow Circle $\frac{\pi}{4}(d_2-d_1^2)$	$\frac{d^2+d_1^2}{16}$	$\frac{\pi(d^4—d_1^4)}{64}$	$\frac{d^4—d_1^4}{10.2d}$	$\frac{\pi(d^4—d_1^4)}{32}$	$\frac{d^4—d_1^4}{5.1d}$
Ellipse $\frac{\pi}{4}ab$	$\frac{a^2}{16}$	$\frac{\pi ba^3}{64}$	$\frac{ba^2}{10.2}$	$\frac{\pi ba^3+ab^3}{64}$	$\frac{ba^3-ab^3}{10.2a}$

Values of I and J for more complicated sections can be worked out from those in table.

TABLE V.—FORMULAS FOR LOADED BEAMS.

Beams of Uniform Cross-section.	Maximum Moment M	Maximum Deflection $\triangle$
Cantilever, load at end..............	Wl	$\frac{Wl^3}{3EI}$
Cantilever, uniform load.............	$\frac{Wl}{2}$	$\frac{Wl^3}{8EI}$
Simple beam, load at middle	$\frac{Wl}{4}$	$\frac{Wl^3}{48EI}$
Simple beam, uniform load..	$\frac{Wl}{8}$	$\frac{5Wl^3}{384EI}$
Beam fixed at one end, supported at other, load at middle.........	$\frac{3Wl}{16}$	$\frac{.0182Wl^3}{EI}$
Beam fixed at one end, supported at other, uniform load...........	$\frac{Wl}{8}$	$\frac{.0054Wl^3}{EI}$
Beam fixed at both ends, load at middle..............................	$\frac{Wl}{8}$	$\frac{Wl^3}{192EI}$
Beam fixed at both ends, uniform load..................................	$\frac{Wl}{12}$	$\frac{Wl^3}{384EI}$
Beam fixed at both ends, load at one end, (pulley arm)............	$\frac{Wl}{2}$	$\frac{Wl^3}{12EI}$

The maximum deflection of cantilevers and beams of uniform strength is greater than when the cross-section is uniform, fifty per cent. greater if the breadth varies, and one hundred per cent greater if the depth varies.

Chapter 2.

FRAME DESIGNS.

5. **General Principles.** The working or moving parts should be designed first and the frame adapted to them.

The moving parts can be first arranged to give the motions and velocities desired, special attention being paid to compactness and to the convenience of the operator.

Novel and complicated mechanisms should be avoided and the more simple and well tried devices used.

Any device which is new should be first tried in a working model before being introduced in the design.

The dimensions of the working parts for strength and stiffness must next be determined and the design for the frame completed. This may involve some modification of the moving parts.

In designing any part of the machine, the metal must be put in the line of stress and bending avoided as far as possible.

Straight lines should be used for the outlines of pieces exposed to tension or compression, circular cross sections for all parts in tortion, and curves of uniform stress for pieces subjected to bending action.

Superfluous metal must be avoided and this excludes all ornamentation as such. There should be a good practical reason for every pound of metal in the machine. It may be sometimes necessary to waste metal in order to save labor in finishing, and in general the aim should be to save labor at the expense of the stock.

It is thus necessary for the designer to be familiar with all the shop processes as well as the principles of

strength and stability. The usual tendency in design, especially of cast iron work, is towards unnecessary weight.

All corners should be rounded for the comfort and convenience of the operator, no cracks or sharp internal angles left where dirt and grease may accumulate, and in general special attention should be paid to so designing the machine that it may be safely and conveniently operated, that it may be easily kept clean, and that oil holes are readily accessible. The appearance of a machine in use is a key to its working condition.

Polished metal should be avoided on account of its tendency to rust, and neither varnish nor bright colors tolerated. The paint should be of some neutral tint and have a dead finish so as not to show scratches or dirt.

Beauty is an element of machine design, but it can only be attained by legitimate means which are appropriate to the material and the surroundings.

Beauty is a natural result of correct mechanical construction but should never be made the object of design.

Harmony of design may be secured by adopting one type of cross-section and adhering to it throughout, never combining cored or box sections with ribbed sections. In cast pieces the thickness of metal should be uniform to avoid cooling strains, and for the same reason sharp corners should be absent. When apertures are cut in a frame either for core-prints or for lightness, the hole or aperture should be the symetrical figure, and not the metal that surrounds it, to make the design pleasing to the eye.

Machine design has been a process of evolution. The earlier types of machines were built before the general introduction of cast iron frames and had frames made of wood or stone, paneled, carved and decorated as in cabinet or architectural designs.

When cast iron frames and supports were first introduced they were made to imitate wood and stone

construction, so that in the earlier forms we find panels, moldings, gothic traceries and elaborate decorations of vines, fruit and flowers, the whole covered with contrasting colors of paint and varnished as carefully as a piece of furniture for the drawing-room. Relics of this transition period in machine architecture may be seen in almost every shop. One man has gone down to posterity as actually advertising an upright drill designed in pure Tuscan.

6. Machine Supports. The fewer the number of supports the better. Heavy frames, as of large engines, lathes, planers, etc., are best made so as to rest directly on a masonry foundation. Short frames as those of shapers, screw machines and milling machines, should have one support of the cabinet form. The use of a cabinet at one end and legs at the other is offensive to the eye being inharmonious. If two cabinets are used provision should be made for a cradle or pivot at one end to prevent twisting of the frame by an uneven foundation. The use of intermediate supports is always to be condemned, as it tends to make the frame conform to the inequalities of the floor or foundation on what has been aptly termed the "caterpillar principle".

A distinction must be made between cabinets or supports which are broad at the base and intended to be fastened to the foundation, and legs similar to those of a table or chair. The latter are intended to simply rest on the floor, should be firmly fastened to the machine and should be larger at the upper end where the greatest bending moment will come.

The use of legs instead of cabinets is an assumption that the frame is stiff enough to withstand all stresses that come upon it, unaided by the foundation, and if that is the case intermediate supports are unnecessary.

Whether legs or cabinets are best adapted to a certain machine the designer must determine for himself.

Where two supports or pairs of legs are necessary under a frame, it is best to have them set a certain distance from the ends, and make the overhanging part

of the frame of a parabolic form, as this divides up the bending moment and allows less deflection at the center. Trussing a long cast-iron frame with iron or steel rods is objectionable on account of the difference in expansion of the two metals and the liability of the tension nuts being tampered with by workmen.

The sprawling double curved leg which originated in the time of Louis XIV and which has served in turn for chairs, pianos, stoves and finally for engine lathes is wrong both from a practical and aesthetic standpoint. It is incorrect in principle and is therefore ugly.

Exercise 1.—Apply the foregoing principles in making a written criticism of some engine or machine frame and its supports.

Chapter 3.

STATIONARY MACHINE MEMBERS.

7. Thin Shells. Let Fig. 1 represent a section of a thin shell, like a boiler shell, exposed to an internal pressure of p pounds per sq. inch. Then, if we consider any diameter AB, will the total upward pressure on upper half of the shell balance the total downward pressure on the lower half and tend to separate the shell at A and B by tension.

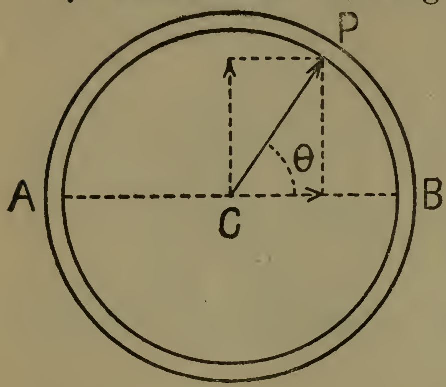


Fig. 1.

Let d = diameter of shell in inches.
r = radius of shell in inches.
l = length of shell in inches.
t = thickness of shell in inches.
S = tensile strength of material.

Draw the radial line CP to represent the pressure on the element P of the surface.

Area of element at P $= lrd\theta$

Total pressure on element $= plrd\theta$.

Vertical pressure on element $= plr \sin \theta d\theta$.

Total vertical pressure on APB $= \int_{\pi}^{0} plr \sin \theta d\theta = 2plr$

The area to resist tension at A and B = 2tl and its total strength = 2tlS.

Equating the pressure and the resistance

$$2tlS = 2plr$$

$$t = \frac{pr}{S} = \frac{pd}{2S} \quad \text{.................. ....(13)}$$

The total pressure on the end of cylinder $=\pi r^2 p$ and the resistance of a circular ring of metal to this pressure $=2\pi rtS$

$$2\pi rSt=\pi r^2 p$$

$$t=\frac{pr}{2S}=\frac{pd}{4S} \quad \text{.........................} (14)$$

Therefore a shell is twice as strong in this direction as in the other. Notice that this same formula would apply to spherical shells.

In calculating the pressure due to a head of water equals h, the following formula is useful:

$$p=0.434h \quad \text{..............................} (15)$$

EXAMPLES.

1. A cast-iron water pipe is 12 inches in diameter and the metal is .45 inches thick. What would be the factor of safety, with an internal pressure due to a head of water of 250 feet?

2. What would be the stress caused by bending due to weight, if the pipe in Ex. 1 were full of water and 24 feet long, the ends being merely supported?

3. A standard lap-welded steam pipe, 8 inches in nominal diameter is 0.32 inches thick and is tested with an internal pressure of 500 pounds per sq. inch. What is the bursting pressure and what is the factor of safety above the test pressure, assuming $S=40000$?

7. **Thick Shells.** There are several formulas for thick cylinders and no one of them is entirely satisfactory. It is however generally admitted that the tensile stress in such a cylinder caused by internal pressure is greatest at the inner circumference and diminishes according to some law from there to the exterior of the shell. This law of variation is expressed differently in the different formulas.

Barlow's Formulas. Here the cylinder diameters are assumed to increase under the pressure, but in such a way that the volume of metal remains constant. Experiment has proved that in extreme cases this last assumption is incorrect. Within the limits of ordinary

TABLE VI.—WROUGHT IRON WELDED TUBES.

For Steam, Gas or Water.

⅛ to ½ inclusive, butt-welded, tested to 300 lbs. per sq. inch hyraulic pressure.

1¼ inch and upwards, lap-welded, tested to 500 lbs. per sq. inch hydraulic pressure.

Nominal size.	Outside Diam. Standard.	Inside Diam. Standard.	Weight per foot. Lbs.	Threads to inch of Screw.	Inside Area. Sq. inches.
⅛	.40	.27	.24	27	.0572
¼	.54	.36	.42	18	.1018
⅜	.67	.49	.56	18	.1886
½	.84	.62	.85	14	.3019
¾	1.05	.82	1.12	14	.5281
1	1.31	1.04	1.67	11½	.8495
1¼	1.66	1.38	2.25	11½	1.4956
1½	1.90	1.61	2.69	11½	2.0358
2	2.37	2.06	3.66	11½	3.3329
2½	2.87	2.46	5 77	8	4.7329
3	3.50	3.06	7.54	8	7.3529
3½	4 00	3.54	9.05	8	9.8423
4	4.50	4.02	10 72	8	12.6924
4½	5.00	4.50	12.49	8	15.9043
4	5.56	5.04	14 56	8	19 9504
6	6.62	6 06	18.77	8	28.8426
7	7.62	7.02	23 41	8	38 7048
8	8.62	7.98	28.35	8	50.0146
9	9.68	9.00	34.07	8	63.6174
10	10.75	10.01	40.64	8	80.1186

practice it is, however, approximately true.

Let d_1 and d_2 be the interior and exterior diameters in inches and let $t = \frac{d_2 - d_1}{2}$ be the thickness.

Let l be the length of cylinder in inches.

Let S_1 and S_2 be the stresses in lbs. per sq. inch at inner and outer circumferences.

Then it may be proved that

$$\frac{S_1}{S_2} = \frac{d_2^2}{d_1^2} \qquad \text{(a)}$$

or the stresses vary inversely as the squares of the corresponding diameters.

Integrating, the total stress on the area 2tl is found to be

$$P = 2 S_1 l \frac{d_1 t}{d_1 + 2t} \qquad \text{(b)}$$

Equating this to the pressure which tends to produce rupture, pdl, where p is the internal unit pressure, there results: * $p = \frac{2S_1 t}{d_1 + 2t}$ .... (16)

Lame's Formula.—In this discussion each particle of the metal is supposed to be subjected to radial compression and to tangential and longitudinal tension and to be in equilibrium under these stresses.

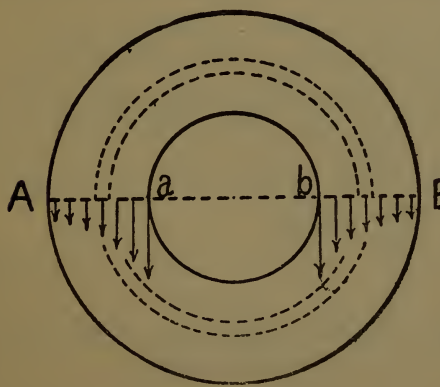


Fig. 2.

Using the same notation as in previous formula:

$$S_1 = \frac{d_2^2 + d_1^2}{d_2^2 - d_1^2} p_1 \qquad \text{(17)}$$

for the maximum stress at the interior.

and $S_2 = \frac{2 d_1^2}{d_2^2 - d_1^2} p_1$ ..(18)

for the stress at the outer surface. †

Fig. 2 illustrates the variation in S from inner to outer surface.

For discussion see Merriman's Mechanics of Materials: * p. 26; † pp. 310–14.

Solving for d_2 in (18) we have

$$d_2=d_1\sqrt{\frac{S_1+p_1}{S_1-p_1}} \quad\text{..................(19)}$$

EXAMPLES.

1. A hydraulic cylinder has an inner diameter of 8 inches, a thickness of four inches and an internal pressure of 1500 lbs. per sq. in. Determine the maximum stress on the metal by Barlow's and Lame''s formulas.

2. Design a cast iron cylinder 6 inches internal diameter to carry a working pressure of 1200 lbs. per sq. in. with a factor of safety of 10.

3. A cast iron water pipe is 1 inch thick and 12 inches internal diameter. Required head of water which it will carry with a factor of safety of 6.

8. Steam Cylinders. Cylinders of steam engines can hardly be considered as coming under either of the preceeding heads. On the one hand the thickness of metal is not enough to insure rigidity as in hydraulic cylinders, and on the other the nature of the metal used, cast iron, is not such as to warrant the assumption of flexibility, as in a thin shell. Most of the formnlas used for this class of cylinder are empirical and founded on modern practice.

Van Buren's formula for steam cylinders is :

$$* \ .0001\,pd+.15\sqrt{d} \quad\text{...............(20)}$$

A formula which the writer has developed is somewhat similar to Van Buren's.

Let s'=tangential stress due to internal pressure.

Then by equation for thin shells

$$s'=\frac{p\,d}{2t}$$

Let s'' be an additional tensile stress due to distortion of the circular section at any weak point.

Then if we regard one-half of the circular section as a beam fixed at A and B (Fig. 3) and assume the

* See Whitham's "Steam Engine Design", p. 27.

maximum bending moment as at C some weak point,

Fig. 3.

the tensile stress on the outer fibres at C due to the bending will be proportional to $\frac{pd^2}{t^2}$ by the laws of flexure, or

$$s''=\frac{cpd^2}{t^2}$$

where c is some unknown constant.

The total tensile stress at C will then be

$$S=s'+s''=\frac{pd}{2t}+\frac{cpd^2}{t^2}$$

Solving for c $\quad c=\frac{st^2}{pd^2}-\frac{t}{2d}$ .... (a)

Solving for t $\quad t=\frac{pd}{4S}+\sqrt{\frac{cpd^2}{S}+\frac{p^2d^2}{16S^2}}$ .... (21)

a form which reduces to that of equation (13) when $c=0$.

An examination of several engine cylinders of standard manufacture shows values of c ranging from .03 to .10, with an average value :

$$c=.06$$

The formula proposed by Professor Barr, in his recent paper on * "Current Practice in Engine Proportions", as representing the average practice among builders of low speed engines is :

$$t=.05\,d+.3 \text{ inch} \quad\ldots\ldots\ldots\ldots\ldots\ldots(22)$$

Experiments made at the Case School of Applied Science in 1896–97 throw some light on this subject. Cast iron cylinders similar to those used on engines were tested to failure by water pressure. The cylinders varied in diameter from six to twelve inches and in thickness from one-half to three-quarters inches.

Contrary to expectations most of the cylinders

* Transactions. A. S. M. E., vol. xviii, p. 741.

failed by tearing around a circumference just inside the flange.

In Table VII are assembled the results of the various experiments for comparison. The values of S by formula (13) are calculated for each cylinder, and by formula (14) for all those which failed on a circumference. It will be noticed that six out of nine failed in the latter way.

TABLE VII.

No.	Diam. d	Pressure. p	Thickness. t	Line of Failure.	Formulas Used. 13 $S=\frac{pd}{2t}$	14 $S=\frac{pd}{4t}$	a c=	Strength of Test-bar.
a	12.16	800	.70	Circum.	6940	3470	.046	18000 lbs.
d	12.45	700	.56	Longi.	7780	—	.047	24000 lbs.
e	9.12	1325	.61	Circum.	9900	4950	.048	24000 lbs.
f	6.12	2500	.65	Circum.	11800	5900	.055	24000 lbs.
1	9.58	600	.402	Longi.	7150	—	.049	24000 lbs.
2	9.375	1050	.573	Circum.	8590	4300	.055	24000 lbs.
3	9.13	975	.596	Circum.	7470	3740	.072	24000 lbs.
4	12.53	700	.571	Longi.	7680	—	.048	24000 lbs.
5	12.56	875	.531	Circum.	10350	5180	.028	24000 lbs.
							Average of c = .05.	

This appears to be due to two causes. In the first place, the influence of the flanges extended to the center of the cylinder, stiffening the shell and preventing the splitting which would otherwise have occurred. In the second place, the fact that the flanges were thicker than the shell caused a zone of weakness near the flange due to shrinkage in cooling, and the presence of what founders call "a hot spot".

The stresses figured from formula (14) in the cases where the failure was on a circumference, are from one-fifth to one-sixth the tensile strength of the test bar.

The strength of a chain is the strength of the weakest link, and when the tensile stress exceeded the strength of the metal near some blow hole or "hot spot", tearing began there and gradually extended around the circumference.

Values of c as given by equation (a) have been calculated for each cylinder, and agree very well except in numbers 3 and 5.

To the criticism that most of the cylinders did not fail by splitting, and that therefore formulas (a) and (21) are not applicable, the answers would be that the chances of failure in the two directions seem about equal, and consequently we may regard each cylinder as about to fail by splitting under the final pressure.

If we substitute the average value of $c=.05$ and a safe value of $s=2000$, formula (21) reduces to:

$$t=\frac{pd}{8000}+\frac{d}{200}\sqrt{p+\frac{p_2}{1600}} \quad \ldots\ldots\ldots\ldots (23)$$

In Kent's Mechanical Engineer's Pocket Book p. 794, the following formula is given as representing closely existing practice:

$$t=.0004\,dp+0.3 \text{ inch} \quad \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

This corresponds to Barr's formula if we take $p=125$ pounds per square inch.

EXAMPLES.

1. Referring to Table VII, verify in at least three experiments the values of S and c as there given.

2. The steam cylinder of a Baldwin locomotive is 22 ins. in diameter and 1.25 ins. thick. Assuming 125 lbs. gauge pressure, find the value of c. Calculate thickness by Van Buren's and Barr's formulas.

3. Determine proper thickness for cylinder of cast iron, if the diameter is 38 inches and the steam pressure 100 lbs. by formulas 13, 20, 21, 23 and 24.

9. Thickness of Flat Plates. An approximate formula for the thickness of flat cast-iron plates may be derived as follows:

Let l = length of plate in inches.
b = breadth of plate in inches.
t = thickness of plate in inches.
p = intensity of pressure in pounds.
S = modulus of rupture lbs. per sq. in.

Suppose the plate to be divided lengthwise into flat strips an inch wide l inches long, and suppose that a fraction p' of the whole pressure causes the bending of these strips.

Regarding the strips as beams with fixed ends and uniformly loaded:

$$S=\frac{6M}{bh^2}=\frac{6\,Wl}{12bh^2}=\frac{p'l^2}{2t^2}$$

and the thickness necessary to resist bending is:

$$t=l\sqrt{\frac{p'}{2S}} \quad \text{.........................(a)}$$

In a similar manner, if we suppose the plate to be divided into transverse strips an inch wide and b inches long, and suppose the remainder of the pressure $p-p'$ equals p'' to cause the bending in this direction, we shall have:

$$t=b\sqrt{\frac{p''}{2S}} \quad \text{.........................(b)}$$

But as all these strips form one and the same plate

the ratio of p' to p'' must be such that the deflection at the center of the plate may be the same on either supposition. The general formula for deflection in this case is

$$\lambda = \frac{Wl^3}{384\ EI}$$

and $I = \frac{t^3}{12}$ for each set of strips. Therefore the deflection is proportional to $\frac{p'l^4}{t^3}$ and $\frac{p''b^4}{t^3}$ in the two cases.

$$\therefore \quad p'l^4 = p''b^4$$

But $$' + p'' = p$$

Solving in these equations for p' and p''

$$p' = \frac{pb^4}{l^4 + b^4}$$

$$p'' = \frac{pl^4}{l^4 + b^4}$$

Substituting these values in (a) and (b):

$$t = lb^2 \sqrt{\frac{p}{2S(l^4 + b^4)}} \qquad \text{(25)}$$

$$t = bl^2 \sqrt{\frac{p}{2S(l^4 + b^4)}} \qquad \text{(26)}$$

As $l > b$ usually, equation (26) is the one to be used. If the plate is square $l = b$ and

$$t = \frac{b}{2} \sqrt{\frac{p}{S}} \qquad \text{(27)}$$

If the plate is merely supported at the edges then formulas (25) and (26) become:

For rectangular plate:

$$t = \frac{bl^2}{2} \sqrt{\frac{3\ p}{S(l^4 + b^4)}} \qquad \text{(28)}$$

For square plate:

$$t = \frac{b}{2} \sqrt{\frac{3\ p}{2S}} \qquad \text{(29)}$$

Formulas for the thickness of a flat plate under a concentrated load at the center, can be derived in a similar manner. A round plate may be treated as square, with side=diameter, without sensible error.

The preceding formulas can only be reparded as approximate. Grashof has investigated this subject and developed rational formulas but his work is too long and complicated for introduction here. His formulas for round plates are as follows :

Round plates :

Supported at edges :

$$t=\frac{d}{2}\sqrt{\frac{5p}{6S}} \qquad \text{(30)}$$

Fixed at edges :

$$t=\frac{d}{2}\sqrt{\frac{2p}{3S}} \qquad \text{(31)}$$

where t and p are the same as before, d is the diameter in inches and S is the safe tensile strength of the material.

Comparing these formulas with (27) and (29) for square plates, they are seen to be nearly identical.

Experiments made at the Case School of Applied Science in 1896–97 on rectangular cast iron plates with load concentrated at the center gave results as follows: Twelve rectangular plates planed on one side and each having an unsupported area of ten by 15 inches were broken by the application of a circular steel plunger one inch in diameter at the geometrical center of each plate. The plates varied in thickness from one-half inch to one and one-eighth inches. Numbers 1 to 6 were merely supported at the edges, while the remaining six were clamped rigidly at regular intervals around the edge.

To determine the value of S, the modulus of rupture of the material, pieces were cut from the edge of the plates and tested by cross-breaking. The average value of S from seven experiments was found to be 33000 lbs. per sq. in.

In Table VIII are given the values obtained for the breaking load W under the different conditions.

TABLE VIII.

Cast iron plates 10x15 ins.

No.	Thickness. t	Breaking Load. W
1	.562	7500
2	.641	11840
3	.745	14800
4	.828	21900
5	1.040	31200
6	1.120	31800
7	.481	9800
8	.646	17650
9	.769	26400
10	.881	33400
11	1.020	47200
12	1.123	59600

Those plates which were merely supported at the edges broke in three or four straight lines radiating from the center. Those fixed at the edges broke in four or five radial lines meeting an irregular oval inscribed in the rectangle. Number 12 however failed by shearing, the circular plunger making a circular hole in the plate with several radial cracks.

EXAMPLES.

1. Calculate the thickness of a steam-chest cover 8×12 inches to sustain a pressure of 90 lbs. per sq. inch with a factor of safety = 10.

2. Calculate the thickness of a circular manhole cover of cast-iron 18 inches in diameter to sustain a pressure of 150 lbs. per sq. inch with a factor of safety = 8, regarding the edges as merely supported.

3. Work out formulas for a rectangular plate having a concentrated load = W at the center, and with edges either supported or fixed.

4. Test values for W given in Table VIII by formulas obtained in example 3.

5. In experiments on steam cylinders, a head 12 inches in diameter and 1.18 inches thick failed under a pressure of 900 lbs. per sq. in. Determine the value of S by formula (31).

10. Machine Frames. For general principles of frame design the reader is referred to Chapter 2. Cast iron is the material most used but steel castings are now becoming common in situations where the stresses are unusually great, as in the frames of presses, shears and rolls for shaping steel.

Cored vs. Rib Sections. Formerly the flanged or rib section was used almost exclusively, as but a few castings were made from each pattern and the cost of the latter was a considerable item. Of late years the use of hollow sections has become more common; the patterns are more durable and more easily molded than those having many projections and the frames when finished are more pleasing in appearance.

The first cost of pattern for hollow work, including the cost of the core-box, is sometimes considerably more but the pattern is less likely to change its shape and in these days of many castings from one pattern, this latter point is of more importance. Finally it may be said that hollow sections are usually stronger for the same weight of metal than any that can be shaped from webs and flanges.

Resistance to Bending. Most machine frames are exposed to bending in one or two directions. If the section is to be ribbed it should be of the form shown in Fig. 4. The metal being of nearly uniform thickness and the flange which is in tension having an area three or four times that of the compression flange. In a steel casting these may be more nearly equal. The hollow section may be of the shape shown in Fig. 5, a hollow rectangle with

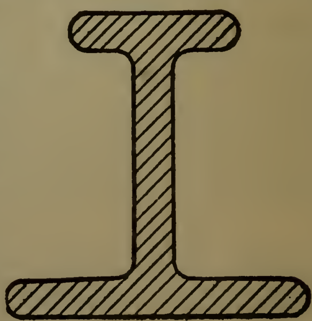


Fig. 4.

the tension side re-enforced and slightly thicker than the other three sides. The re-enforcing flanges at A and B may often be utilized for the attaching of other members to the frame as in shapers or drill presses. The box section has one great advantage over the I section in that its moment of resistance to side bending or to twisting is usually much greater. The double I or the **U** section is common where it is necessary to have two parallel ways for sliding pieces as in lathes and planers. As is shown in Fig. 6 the two Is are usually connected at intervals by cross girts.

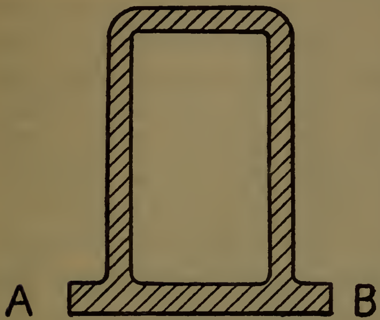


Fig. 5.

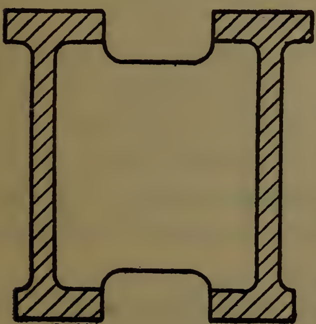


Fig. 6.

Besides making the cross-section of the most economical form, it is often desirable to have such a longitudinal profile as shall give a uniform fibre stress from end to end. This necessitates a parabolic or elliptic outline of which the best instance is the housing or upright of a modern iron planer.

Resistance to Twisting. The hollow circular section is the ideal form for all frames or machine members which are subjected to torsion. If subjected also to bending the section may be made elliptical or, as is more common, thickened on two sides by making the core oval. See Fig. 7. As has already been pointed out the box sections are in general better adapted to resist twisting than the ribbed or I sections.

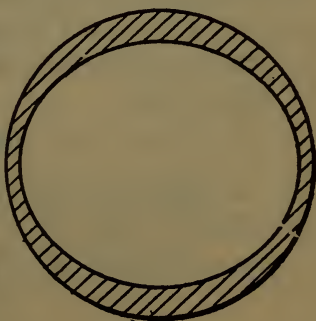


Fig. 7.

Frames of Machine Tools. The beds of lathes are subjected to bending on account of their own weight and that of the saddle and on account of the downward pressure on the tool when work is being turned. They are usually subjected to torsion on account of the uneven pressure of the supports. The box section is then the best; the double I commonly used is very weak against twisting. The same principle would apply in designing the beds of planers but the usual method of driving the table by means of a gear and rack prevents the use of the box section. The uprights of planers and the cross rail are subjected to severe bending moments and should have profiles of uniform strength. The uprights are also subject to side bending when the tool is taking a heavy side cut near the top. To provide for this the uprights may be of a box section or may be reinforced by outside ribs.

The upright of a drill press or vertical shaper is exposed to a constant bending moment equal to the upward pressure on the cutter × the distance from center of cutter to center of upright. It should then be of constant cross-section from the bottom to the top of the straight part. The curved or goose-necked portion should then taper gradually.

The frame of a shear press or punch is usually of the G shape in profile with the inner fibers in tension and the outer in compression. The cross-section should be as in Fig. 4 or Fig. 5, preferably the latter and should be graduated to the magnitude of the bending moment at each point.

EXERCISES.

1. Discuss the stresses and the arrangement of material in the girder frame of a Corliss engine.

2. Ditto in the G frame of a band saw.

Chapter 4.

11. Spiral Springs. The most common form of spring used in machinery is the spiral or helical spring made of round brass or steel wire. Such springs may be used to resist extension or compression or they may be used to resist a twisting moment.

Tension and Compression.

Let L=length of axis of spring.
D=mean diameter of spring.
l=developed length of wire.
d=diameter of wire.
n=number of coils.
P=tensile or compressive force.
x=corresponding extension or compression.
S=safe torsional or shearing strength of wire.
=2500 for spring brass wire.
=75000 to 115000 for cast steel tempered.
G=modulus of torsional elasticity.
=6000000 for spring brass wire.
=12000000 to 15000000 for cast steel, tempered.

Then $$l=\sqrt{\pi^2 D^2 n^2+L^2}$$

If the spring were extended until the wire became straight it would then be twisted n times, or through an angle$=2\pi n$ and the stretch would be $l-L$.

The angle of torsion for a stretch$=x$ is then

$$\theta=\frac{2\pi nx}{l-L} \quad \text{.................... (a)}$$

Suppose that a force P' acting at a radius $\frac{D}{2}$ will twist this same piece of wire through an angle θ causing a stress S at the surface of the wire. Then will the distortion of the wire per inch of length be $s=\frac{\theta d}{2l}$

and $$S=\frac{5.1T}{d^3}=\frac{5.1P^1D}{2d^3} \quad \text{.................... (b)}$$

$$\therefore G = \frac{S}{s} = \frac{10.2\ P^1 Dl}{2d^4\theta} \qquad \text{(c)}$$

In thus twisting the wire the force required will vary uniformly from o at the beginning to P^1 at the end provided the elastic limit is not passed, and the average force will be

$= \frac{P^1}{2}$ The work done is therefore $\frac{P^1 D\theta}{4}$

If the wire is twisted through the same angle by the gradual application of the direct pressure P, compressing or extending the spring the amount x, the work done will be

$\frac{Px}{2}$ But $\frac{P^1 D\theta}{4} = \frac{Px}{2}$

$$\therefore P^1 = \frac{2Px}{D\theta} \qquad \text{(d)}$$

Substituting this value of P^1 in (c) and solving for x:

$$x = \frac{Gd^4\theta^2}{10.2\ Pl}$$

Substituting the value of θ from (a) and again solving for x :

$$x = \frac{10.2 Pl}{Gd^4} \left\{ \frac{l - L}{2\pi n} \right\}^2 \qquad \text{(e)}$$

If we neglect the original obliquity of the wire then $l = \pi Dn$ and $L = o$ and equation (e) reduces to

$$x = \frac{2.55 PlD^2}{Gd^4} \qquad \text{(32)}$$

Making the same approximation in equation (d) we have $P^1 = P$

i. e. — a force P will twist the wire through approximately the same angle when applied to extend or compress the spring, as if applied directly to twist a piece of straight wire of the same material with a lever arm $= \frac{D}{2}$

This may be easily shown by a model.

The safe working load may be found by solving for P^1 in (b) and remembering that $P=P^1$

$$P=\frac{Sd^3}{2.55\ D} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

when S is the safe shearing strength.

Substituting this value of P in (24) we have for the safe deflection:

$$x=\frac{lDS}{Gd} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (34)$$

12. Square Wire. The value of the stress for a square section is:

$$S=\frac{4.24T}{d^3}$$

where d is the side of square.

The distortion at the corners caused by twisting through an angle θ is:

$$s=\frac{\theta d}{l\sqrt{2}}$$

Equation (c) then becomes:

$$G=\frac{6P'Dl}{2d^4\theta}$$

The three principal equations (32), (33) and (34) then reduce to:

$$x=\frac{1.5PlD^2}{Gd^4} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (35)$$

$$P=\frac{Sd^3}{2.12D} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (36)$$

$$x=\frac{lDS}{Gd\sqrt{2}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

The square section is not so economical of material as the round.

13. Experiments. Tests made on about 1700 tempered steel springs at the French Spring Works in Pittsburg were reported in 1901 by Mr. R. A French. These were all compression springs of round steel and were given a permanent set before testing by being

closed coil to coil several times. Mr. French as a result of the experiments arrives at the following conclusions:

1. The average value of G is 14,500,000.

2. The safe working stress S depends upon the proportions of the spring and varies from 75000 to 112,000 lbs. per sq. inch for a good grade of steel properly tempered.

3. If $R = \frac{D}{d}$ the ratio of spring diameter to wire diameter, the following values of S may be safely assumed.

VALUES OF S.

	R=3	R=8
d=⅜ inch or less	112,000	85,000
$d = \frac{7}{16}$ inch to ¾ inch..	110,000	80,000
$d = \frac{13}{16}$ inch to 1¼ inch...	105.000	75,000

4. When a spring is subjected to sudden shocks a smaller value of S must be used.

5. In designing close coil extension springs the value of G will be as above but the values of S should not be over two-thirds the corresponding values for compression springs.

14. Spring in Torsion. If a spiral spring is used to resist torsion instead of tension or compression, the wire itself is subjected to a bending moment. We will use the same notation as in the last article, only that P will be taken as a force acting tangentially to the circumference of the spring at a distance $\frac{D}{2}$ from the axis, and S will now be the safe transverse strength of the wire, having the following values:

S=3000 for spring brass wire.
=90,000 to 125000 for cast steel tempered.

$E=9000000$ for spring brass wire.
$=30000000$ for cast steel tempered.

Let $\theta=$ angle through which the spring is turned by P.

The bending moment on the wire will be the same throughout and $=\frac{PD}{2}$ This is best illustrated by a model.

To entirely straighten the wire by unwinding the spring would require the same force as to bend straight wire to the curvature of the helix.

To simplify the equations we will disregard the obliquity of the helix, then will $l=\pi Dn$ and the radius of curvature $=\frac{D}{2}$

Let $M=$ bending moment caused by entirely straightening the wire; then by mechanics

$$M=\frac{EI}{R}=\frac{2EI}{D}$$

and the corresponding angle through which spring is turned is $2\pi n$.

But it is assumed that a force P with a radius $\frac{D}{2}$ turns the spring through an angle θ.

$$\therefore \quad \frac{PD}{2}=\frac{2EI}{D}\times\frac{\theta}{2\pi n}$$

$$=\frac{EI\theta}{\pi Dn}=\frac{EI\theta}{l}$$

Solving for θ:

$$\theta=\frac{PDl}{2EI} \quad \text{.......................(a)}$$

and if wire is round

$$\theta=\frac{10.2PDl}{Ed^4} \quad \text{.......................(38)}$$

The bending moment for round wire will be

$$\frac{PD}{2}=\frac{Sd^3}{10.2} \quad \text{.......................(39)}$$

and this will also be the safe twisting moment that can be applied to the spring when $S =$ working strength of wire. The safe angle of deflection is found by substituting this value of $\frac{PD}{2}$ in (38):

Reducing: $$\theta = \frac{2lS}{Ed} \qquad (40)$$

15. **Flat Springs.** Ordinary flat springs of uniform rectangular cross-section can be treated as beams and their strength and deflection calculated by the usual formulas.

In such a spring the bending and the stress are greatest at some one point and the curvature is not uniform.

To correct this fault the spring is made of a constant depth but varying width.

If the spring is fixed at one end and loaded at the other the plan should be a triangle with the apex at the loaded end. If it is supported at the two ends and loaded at the center, the plan should be two triangles with their bases together under the load forming a rhombus. The deflection of such a spring is one and a half times that of a rectangular spring.

As such a spring might be of an inconvenient width, a compound or leaf-spring is made by cutting the triangular spring into strips parallel to the axis, and piling one above another as in Fig. 8.

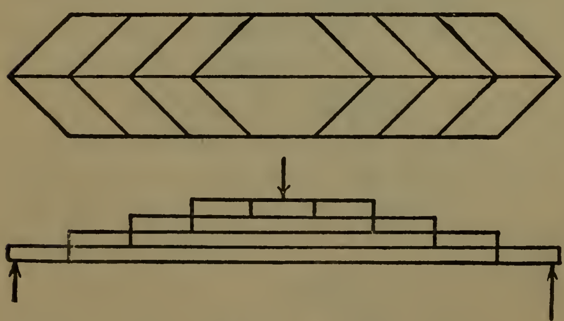


Fig. 8.

This arrangement does not change the principle, save that the friction between the leaves may increase the resistance somewhat.

Let l=length of span.
b=breadth of leaves.
t=thickness of leaves.
n=number of leaves.
W=load at center.
$\triangle$=deflection at center.

S and E may be taken as 80000 and 30000000 respectively.

Strength :

$$M=\frac{Wl}{4}=\frac{Snbt^2}{6}$$

$$W=\frac{2}{3}\,\frac{Snbt^2}{l}\ldots\ldots\ldots\ldots\ldots\ldots(41)$$

Elasticity :

$$\triangle=\frac{Wl^3}{32EI}\text{ where }I=\frac{nbt^3}{12}$$

$$\therefore\quad\triangle=\frac{3Wl^3}{8Enbt^3}\ldots\ldots\ldots\ldots\ldots\ldots(42)$$

For the benefit of those who wish to design springs in quantity, reference is made to Trans. Am. Soc. Mech. Eng. Vol. XVII. p. 340, where will be found very complete tables for both helical and flat springs.

EXAMPLES.

1. A spring balance is to weigh 25 pounds with an extension of 2 inches, the diameter of spring being ⅝ inches and the material, tempered steel.

Determine the diameter and length of wire, and number of coils.

2. Determine the safe twisting moment and angle of torsion for the spring in example 1, if used for a torsional spring.

3. Design a compound flat spring for a locomotive to sustain a load of 16000 lbs. at the center, the span being 40 inches, the number of leaves 12 and the material steel.

4. Determine the maximum deflection of the above spring, under the working load.

5. Measure various indicator springs and determine value of G from rating of springs.

6. Measure various brass extension springs calculate safe static load and safe stretch.

7. Make an experiment on torsion spring to determine distortion under a given load and calculate value of E.

Chapter 5.

FASTENINGS.

16. Bolts and Nuts. Tables of dimensions for U. S. standard bolts and nuts are to be found in all hand books and will not be repeated here.

Roughly the diameter at root of thread is .83 o. the outer diameter in this system, and the pitch in inches is given by the formula

$$p = .24\sqrt{d+.625} - .175 \qquad (43)$$

where d = outer diameter.

In designing bolts to resist simple tension, calculate the area needed to resist the given tension, divide this by the number of bolts to be used and the quotient will be the area of one bolt at the root of thread.

From the tables the corresponding diameter and the diameter of body of bolt can be determined.

Bolts may be divided into three classes which are given in the order of their merit.

1. Through bolts, having a head on one end and a nut on the other.

2. Stud bolts, having a nut on one end and the other screwed into the casting.

3. Tap bolts or screws having a head at one end and the other screwed into the casting.

The principal objection to the last two forms and especially to (3) is the liability of sticking or breaking off in the casting.

Any irregularity in the bearing surfaces of head or nut where they come in contact with the casting, causes a bending action and consequent danger of rupture.

17. Eye Bolts and Hooks. In designing eye bolts it is customary to make the combined sectional area of the sides of the eye one and one half-times that of

TABLE IV.—SAFE WORKING STRENGTH OF WROUGHT IRON BOLTS.

Diam of Bolt. — Inch.	Diam. at Root of Thread. Inches.	Area at Root of Thread. Sq. Ins.	Safe Load in Tension — Lbs.	Safe Load in Shear. — Lbs.	Thr'ds per Inch. — No.
¼	.185	.0269	150	220	20
5/16	.240	.0452	250	350	18
⅜	.294	.0679	375	500	16
7/16	.344	.0930	510	675	14
½	.400	.1257	690	880	13
9/16	.454	.162	890	1120	12
⅝	.507	.202	1110	1380	11
¾	.620	.302	1660	2000	10
⅞	.731	.420	2310	2700	9
1	.837	.550	3025	3535	8
1⅛	.940	.694	3815	4475	7
1¼	1.065	.891	4900	5520	7
1⅜	1.160	1.057	5815	6680	6
1½	1.284	1.295	7125	7950	6
1⅝	1.389	1.515	8335	9330	5½
1¾	1.490	1.744	9590	10825	5
1⅞	1.615	2.049	11270	12425	5
2	1.712	2.302	12660	14130	4½

the bolt to allow for obliquity and an uneven distribution of stress.

Large hooks should be designed to resist combined bending and tension; the bending moment is equal to the load × the longest perpendicular from the center line of hook to line of load.

Check Nuts: A check is a thin nut screwed firmly against the main nut to prevent its working loose, and is commonly placed outside.

As the whole load is liable to come on the outer nut, it would be more correct to put the main nut outside.

After both nuts are firmly screwed down, the outer one should be held stationary and the inner one reversed against it with what force is deemed safe, that the greater reaction may be between the nuts.

The foregoing table is convenient for determining the size of bolt needed to resist tension or shear and is based on the U. S. standard form of thread using a factor of safety = 10.

For steel bolts, increase the loads given in the table 20 per cent. The loads given are correct within 10 pounds. The shearing area used is that of the body of the bolt.

EXAMPLES.

1. Discuss the effect of the initial tension caused by the screwing up of the nut on the bolt, in the case of steam fittings, etc.; *i. e.* should this tension be added to the tension due to the steam pressure, in determining the proper size of bolt?

2. Determine the number of ¾ inch bolts necessary to hold on the head of a steam cylinder 15 inches diameter, with the internal pressure 90 pounds per square inch, and factor of safety = 12.

3. Show what is the proper angle between the handle and the jaws of a fork wrench

(1) If used for square nuts:

(2) If used for hexagon nuts; illustrate by figure.

4. Determine the length of nut theoretically necessary to prevent stripping of the thread, in terms of the outer diameter of the bolt.

(1) With U. S. standard thread.
(2) With square thread of same depth.

5. Design a hook with a swivel and eye at the top to hold a load of one ton with a factor of safety = 5, the center line of hook being three inches from line of load, and the material wrought iron.

18. Riveted Joints. No attempt will be made to go into the details of this subject, but only to state the general principles involved in designing joints.

Riveted joints may be divided into two general classes: lap joints where the two plates lap over each other, and butt joints where the edges of the plates butt together and are joined by over-lapping straps or welts. If the strap is on one side only, the joint is known as a butt joint with one strap; if straps are used inside and out the joint is called a butt joint with two straps. Butt joints are generally used when the material is more than one half inch thick.

Any joint may have one, two or more rows of rivets and hence be known as a single riveted joint, a double riveted joint, etc.

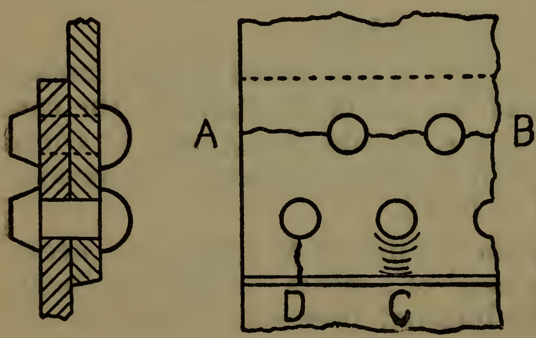


Fig. 9.

Any riveted joint is weaker than the original plate, simply because holes cannot be punched or drilled in the plate for the introduction of rivets without removing some of the metal.

Fig. 9 shows a double riveted lap joint and Fig. 10 a single riveted butt joint with two straps.

Riveted joints may fail in any one of four ways :

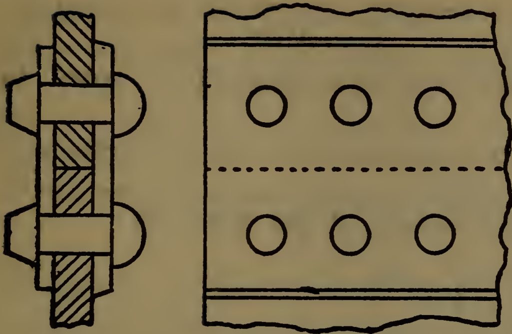


Fig. 10.

1. By tearing of the plate along a line of rivet holes, as at AB, Fig. 9.

2. By shearing of the rivets.

3. By crushing and wrinkling of the plate in front of each rivet as at C, Fig. 9, thus causing leakage.

4. By splitting of the plate opposite each rivet as at D, Fig. 9. The last manner of failure may be prevented by having a sufficient distance from the rivet to the edge of the plate. Practice has shown that this distance should be at least equal to the diameter of a rivet.

Let: t=thickness of plate.
d=diameter of rivet-hole
p=pitch of rivets.
n=number of rows of rivets.
T=tensile strength of plate.
C=bearing or crushing strength of plate.
S=Shearing strength of rivet.

Average values of the constants are as follows:

Material.	T	C	S
Wrought Iron.	50 000	80 000	40 000
Soft Steel.	56 000	90 000	45 000

19. Lap Joints. This division also includes butt joints which have but one strap.

Let us consider the shell as divided into strips at right angles to the seam and each of a width = p. Then the forces acting on each strip are the same and we need to consider but one strip.

The resistance to tearing across of the strip between rivet holes is $(p-d)tT$ ....(a) and this is independent of the number of rows of rivets.

The resistance to compression in front of rivets is

$$ndtC \text{......................................(b)}$$

and the resistance to shearing of the rivets is

$$\frac{\pi}{4} nd^2S \text{............................(c)}$$

The values of the constants given above are only average values and are liable to be modified by the exact grade of material used and the manner in which it is used.

The tensile strength of soft steel is reduced by punching from three to twelve per cent according to the kind of punch used and the width of pitch. The shearing strength of the rivets is diminished by their tendency to tip over or bend if they do not fill the holes, while the bearing or compression is doubtless relieved to some extent by the friction of the joint. The average values given allow roughly for these modifications.

If we call the tensile strength T=unity then the relative values of C and S are 1.6 and 0.8 respectively.

Substituting these relative values of T, C and S in our equations, by equating (b) and (c) and reducing we have

$$d=2.55t \text{..............................(44)}$$

Equating (a) and (c) and reducing we have

$$p=d+.628\frac{nd^2}{t} \text{....................(45)}$$

Or by equating (a) and (b)

$$p=d+1.6nd \text{.......................(46)}$$

These proportions will give a joint of equal strength throughout, for the values of constants assumed.

20. Butt Joints with two Straps. In this case the resistance to shearing is increased by the fact that the

rivets must be sheared at both ends before the joint can give way. Experiment has shown this increase of shearing strength to be about 85 per cent and we can therefore take the relative value of S as 1.5 for butt joints.

This gives the following values for d and p

$$d=1.36t \qquad (47)$$

$$p=d+1.18\frac{nd^2}{t} \qquad (48)$$

$$p=d+1.6nd \qquad (49)$$

In the preceding formulas the diameter of hole and rivet have been assumed to be the same.

The diameter of the cold rivet before insertion will be $\frac{1}{16}$ inches less than the diameter given by the formulas.

Experiments made in England by Prof. Kennedy give the following as the proportions of maximum strength:

Lap joints	$d=2.33t$
	$p=d+1.375nd$
Butt joints	$d=1.8t$
	$p=d+1.55nd$

21. Efficiency of Joints. The efficiency of joints designed like the preceding is simply the ratio of the section of plate left between the rivets to the section of solid plate, or the ratio of the clear distance between two adjacent rivet holes to the pitch. From formula (35) we thus have

$$\text{Efficiency}=\frac{1.6n}{1+1.6n} \qquad (50)$$

This gives the efficiency of single, double and triple riveted seams as

.615, .762 and .828 respectively.

Notice that the advantage of a double or triple riveted seam over the single is in the fact that the pitch bears a greater ratio to the diameter of a rivet, and therefore the proportion of metal removed is less.

22. Butt Joints with unequal Straps. One joint in common use requires special treatment.

It is a double-riveted butt joint in which the inner strap is made wider than the outer and an extra row of rivets added, whose pitch is double that of the original seam; this is sometimes called diamond riveting. See Fig. 11.

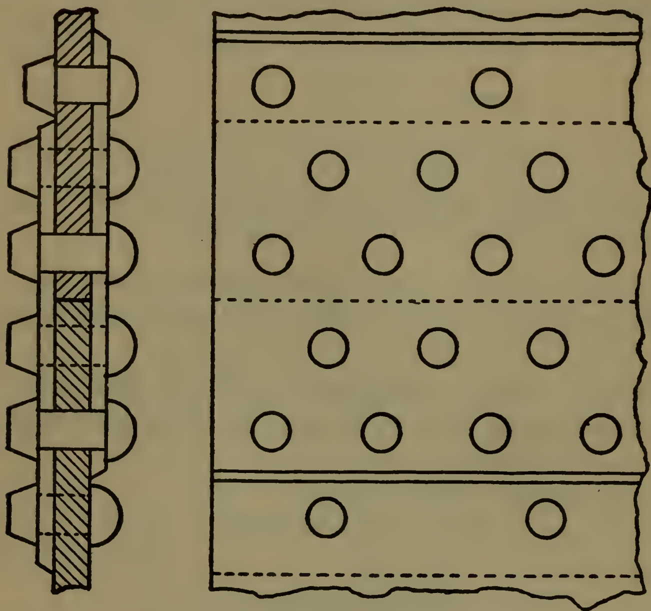


Fig. 11.

This outer row of rivets is then exposed to single shear and the original rows to double shear.

Consider a strip of plate of a width $=2p$. Then the resistance to tearing along the outer row of rivets is

$$(2p-d)tT$$

As there are five rivets to compress in this strip the bearing resistance is

$$5dtC$$

As there is one rivet in single shear and four in double shear the resistance to shearing is

$$\left\{1+(4\times 1.85)\right\}\frac{\pi}{4}d^2S=6.6d^2S$$

Solving these equations as in previous cases, we have for this particular joint

$$d=1.52t \qquad (51)$$

$$2p=9d$$

$$p=4.5d \qquad (52)$$

$$\text{Efficiency}=\frac{2p-d}{2p}=\frac{8}{9} \qquad (53)$$

23. Practical Rules. The formulas given above show the proportions of the usual forms of joints for uniform strength.

In practice certain modifications are made for economic reasons. To avoid great variation in the sizes of rivets the latter are graded by sixteenths of an inch, making those for the thicker plates considerably smaller than the formula would allow, and the pitch is then calculated to give equal tearing and shearing strength.

The following table gives the proportions generally used in this country for lap joints, as given by "Locomotive" 1882.

TABLE X.—RIVETED LAP JOINTS.

Thickness of Plate.	Diam. of Rivet.	Diam. of Hole.	Pitch.		Efficiency.	
			Single.	Double	Single.	Double
¼	⅝	$\frac{11}{16}$	2	3	.66	.77
$\frac{5}{16}$	$\frac{11}{16}$	¾	$2\frac{1}{16}$	3⅛	.64	.76
⅜	¾	$\frac{13}{16}$	2⅛	3¼	.62	.75
$\frac{7}{16}$	$\frac{13}{16}$	⅞	$2\frac{3}{16}$	3⅜	.60	.74
½	⅞	$\frac{15}{16}$	2¼	3½	.58	.73

This table is for iron plates and iron rivets. For steel plates with iron or steel rivets increase the diameter of rivets $\frac{1}{16}$ inch, the pitch remaining the same.

A similar table has been calculated for butt joints. Table XI is for iron plates with iron rivets. For steel plates increase the diameter of rivet $\frac{1}{16}$ inch, the pitch remaining the same.

TABLE XI.—RIVETED BUTT JOINTS.

Thickness of Plate.	Diam. of Rivet.	Diam. of Hole.	Pitch.		
			Single.	Double.	Triple.
½	¾	$\frac{13}{16}$	2⅜	4	5½
⅝	$\frac{13}{16}$	⅞	2⅜	3¾	5¼
¾	⅞	$\frac{15}{16}$	2⅜	3¾	5⅛
⅞	$\frac{15}{16}$	1	2⅜	3¾	5
1	1	$1\frac{1}{16}$	2⅜	3¾	5

EXAMPLES.

1. Investigate proportions of joints for half-inch plate in Table X and criticise.

2. Criticise in same way the proportions of joints for one inch plate in Table XI.

3. Show the effect of increasing the diameter of rivets $\frac{1}{16}$ inch for steel plates and prove by example.

4. A cylindrical boiler 5×16 ft. is to have long seams double-riveted laps and ring seams single riveted laps. If the internal pressure is 90 lbs. gauge pressure and the material soft steel, determine thickness of plate and proportions of joints. Factor of safety to be five and efficiency of joints to be allowed for.

5. A marine boiler is 11 ft. 6 ins. in diameter and 14 ft. long. The long seams are to be diamond riveted butt joints and the ring seams ordinary double riveted butt joints. The internal pressure is to be 175 lbs. gauge and the material is to be steel of 60,000 lbs.

tensile strength. Determine thickness of shell and proportions of joints. Net factor of safety to be five allowing for efficiency of joints.

6. Design a diamond riveted joint such as shown in Fig. 11a for a steel plate ⅝ inches thick. Outer cover plate is ⅝ inches and inner cover plate $\frac{7}{16}$ inches thick. Determine efficiency of joints.

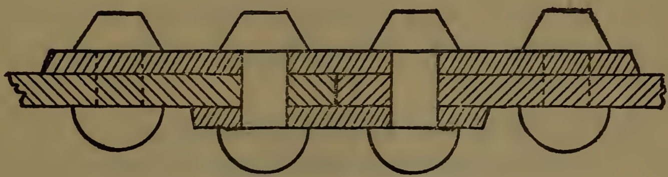


Fig. 11a.

7. The single lap joint with cover plate, as shown in Fig. 12, is to have pitch of outer rivets double that of inner row. Determine diameter and pitch of rivets for ⅜ inch plate and the efficiency of joint.

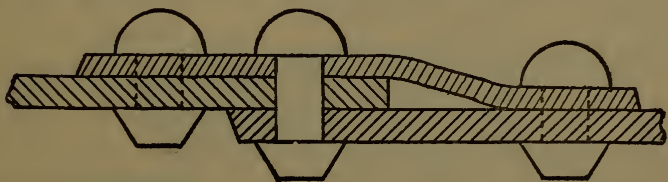


Fig. 12.

24. Joint Pins. A joint pin is a bolt exposed to double shear. If the pin is loose in its bearings it should be designed with allowance for bending, by adding from 30 to 50 per cent to the area of cross-section needed to resist shearing alone. Bending of the pin also tends to spread apart the bearings and this should be prevented by having a head and nut or cotter on the pin.

If the pin is used to connect a knuckle joint as in boiler stays, the eyes forming the joint should have a a net area 50 per cent in excess of the body of the stay, to allow for bending and uneven tension.

25. Cotters. A cotter is a key which passes diametrically through a hub and its rod or shaft, to fasten them together, and is so called to distinguish it from shafting keys which lie parallel to axis of shaft.

Its taper should not be more than 4 degrees or about 1 in 15, unless it is secured by a screw or check nut.

The rod is sometimes enlarged where it goes in the hub, so that the effective area of cross-section where the cotter goes through may be the same as in the body of the rod

Let: d=diameter of body of rod.
d_1=diameter of enlarged portion.
t=thickness of cotter, usually $=\frac{d_1}{4}$
b=breadth of cotter.
l=length of rod beyond cotter.

Suppose that the applied force is a pull on the rod —causing tension on the rod and shearing stress on the cotter.

The effective area of cross section of rod at cotter is

$$\frac{\pi d_1^2}{4} - \frac{d_1^2}{4} = (\pi - 1)\frac{d_1^2}{4}$$

and this should equal the area of cross-section of the body of rod.

$$(\pi - 1)\frac{d_1^2}{4} = \frac{\pi d^2}{4}$$

$$d_1 = d\sqrt{\frac{\pi}{\pi - 1}} = 1.21d \quad \text{.............. (54)}$$

Let P=pull on rod.
S=shearing strength of material.

The area to resist shearing of cotter is

$$2bt = \frac{bd_1}{2} = \frac{P}{S}$$

$$\therefore \quad b = \frac{2P}{d_1 S} \quad \text{..........................(a)}$$

The area to resist shearing of rod is

$$2d_1 l = \frac{P}{S}$$

$$\text{and } l = \frac{P}{2d_1 S} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(b)}$$

If the metal of rod and cotter are the same

$$2d_1 l = \frac{bd_1}{2}$$

$$l = \frac{b}{4} \quad \ldots\ldots\ldots\ldots\ldots\ldots (55)$$

Great care should be taken in fitting cotters that they may not bear on corners of hole and thus tear the rod in two.

A cotter or pin subjected to alternate stresses in opposite directions should have a factor of safety double that otherwise allowed.

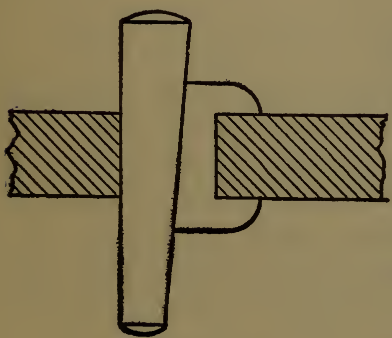


Fig. 13.

Adjustable cotters, used for tightening joints or bearings are usually accompanied by a gib having a taper equal and opposite to that of the cotter. (Fig. 13) In designing these for strength the two can be regarded as resisting shear together.

For shafting keys see chapter on shafting.

EXAMPLES.

1. Design a knuckle joint for a soft steel boiler stay, the pull on stay being 12000 lbs. and the factor of safety, six.

2. Determine the diameter of a round cotter pin for equal strength of rod and pin

3. A rod of wrought iron has keyed to it a piston 18 inches in diameter, by a cotter of machinery steel.

Required the two diameters of rod and dimensions of cotter to sustain a pressure of 150 pounds per square inch on the piston. Factor of safety = 8.

4. Design a cotter and gib for connecting rod of engine mentioned in Ex. 3, both to be of machinery steel and .75 inches thick.

Chapter 6.

SLIDING BEARINGS.

26. Slides in General. The surfaces of all slides should have sufficient area to limit the intensity of pressure and prevent forcing out of the lubricant. No general rule can be given for the limit of pressure. Tool marks parallel to the sliding motion should not be allowed, as they tend to start grooving. The sliding piece should be as long as practicable to avoid local wear on stationary piece and for the same reason should have sufficient stiffness to prevent springing. A slide which is in continuous motion should lap over the guides at the ends of stroke, to prevent the wearing of shoulders on the latter and the finished surfaces of all slides should have exactly the same width as the surfaces on which they move for a similar reason.

Where there are two parallel guides to motion as in a lathe or planer it is better to have but one of these depended upon as an accurate guide and to use the other merely as a support. It must be remembered that any sliding bearing is but a copy of the ways of the machine on which it was planed or ground and in turn may reproduce these same errors in other machines. The interposition of handscraping is the only cure for these hereditary complaints.

In designing a slide one must consider whether it is accuracy of motion that is sought, as in the ways of a planer or lathe, or accuracy of position as in the head of a milling machine. Slides may be divided according to their shapes into angular, flat and circular slides.

27. Angular Slides. An angular slide is one in which the guiding surface is not normal to the direction of pressure. There is a tendency to displacement sideways, which necessitates a second guiding surface inclined to the first. This oblique pressure constitutes

the principal disadvantage of angular slides. Their principal advantage is the fact that they are either self adjusting for wear, as in the ways of lathes and planers, or require at most but one adjustment.

Fig. 14 shows one of the V's of an ordinary planing machine. The platen is held in place by gravity. The angle between the two surfaces is usually 90° but may be more in heavy machines. The grooves g, g are intended to hold the oil in place; oiling is sometimes effected by small rolls recessed into the lower piece and held against the platen by springs.

The principal advantage of this form of way is its ability to hold oil and the great disadvantage its faculty for catching chips and dirt.

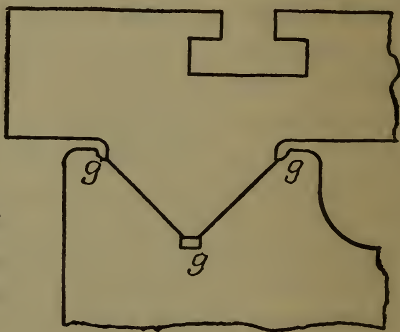


Fig. 14.

Fig. 15 shows an inverted V such as is common on the ways of engine lathes. The angle is about the same as in the preceding form but the top of the V should be rounded as a precaution against nicks and bruises.


Fig. 15.

The inverted V is preferred for lathes since it will not catch dirt and chips. It needs frequent lubrication as the oil runs off rapidly. Some lathe carriages are provided with extensions filled with oily felt or waste to protect the ways from dirt and keep them wiped and oiled. Side pressure tends to throw the carriage from the ways; this action may be prevented by a heavy weight hung on the carriage or by gibbing the carriage at the back (See Fig.20).

The objection to this latter form of construction is the fact that it is practically impossible to make and keep the two V's and the gibbed slide all parallel.

28. Gibbed Slides. All slides which are not self-adjusting for wear must be provided with gibs and adjusting screws. Fig. 16 shows the most common form as used in tool slides for lathes and planing machines.

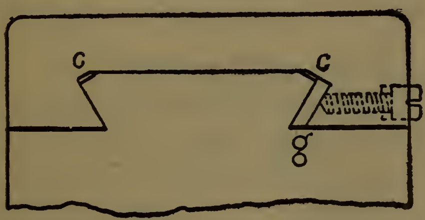


Fig. 16.

The angle employed is usually 60°; notice that the corners c c are clipped for strength and to avoid a corner bearing; notice also the shape of gib. It is better to have the points of screws coned to fit gib and *not* to have flat points fitting recesses in gib. The latter form tends to spread joint apart by forcing gib down. If the gib is too thin it will spring under the screws and cause uneven wear.

The cast iron gib, Fig. 17, is free from this latter defect but makes the slide rather clumsy. The screws however are more accessible in this form. Gibs are sometimes made slightly tapering and adjusted by a screw and nut giving endwise motion.

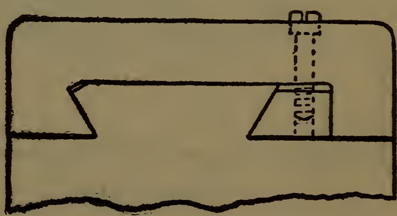


Fig. 17.

29. Flat Slides. This type of slide requires adjustment in two directions and is usually provided with gibs and adjusting screws. Flat ways on machine tools are the rule in English practice and are gradually coming into use in this country. Although more expensive at first and not so simple they are more durable and usually more accurate than the angular ways.

Fig. 18 illustrates a flat way for a planing machine. The other way would be similar to this but without adjustment. The normal pressure and the friction are less than with angular ways and no amount of side pressure will lift the platen from its position.

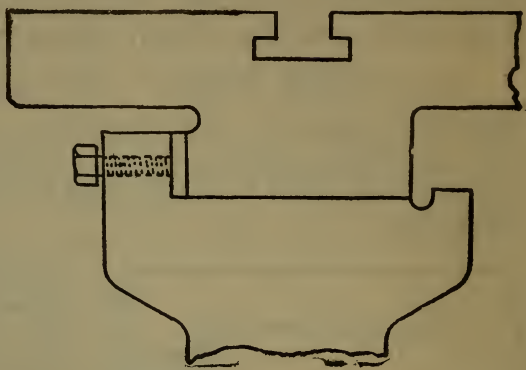


Fig. 18.

Fig. 19 shows a portion of the ram of a shaping machine and illustrates the use of an L gib for adjustment in two directions. Fig. 20 shows a gibbed slide for holding down the back of a lathe carriage with two adjustments.

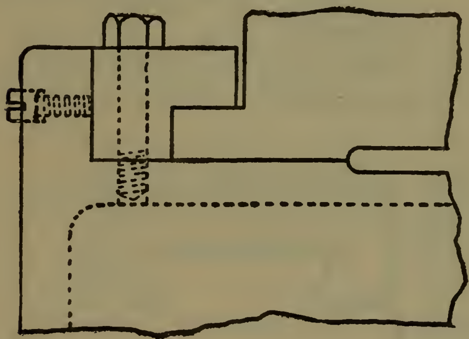


Fig. 19.

The gib g is tapered and adjusted by a screw and nuts. The saddle of a planing machine or the table of a shaper usually has a rectangular gibbed slide above and a taper slide below, this form of the upper slide being necessary to hold the weight of the overhanging metal. Some lathes and planers are built with one V or angular way for guiding the carriage or platen and one flat way acting merely as a support.

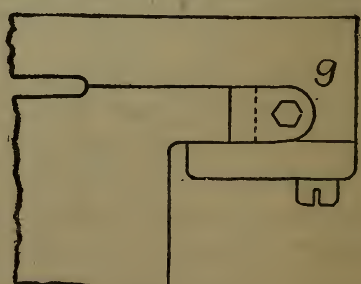


Fig. 20.

30. Circular Guides. Examples of this form may be found in the column of the drill press and the overhanging arm of the milling machine. The cross heads of steam engines are sometimes fitted with circular guides; they are more frequently flat or angular. One advantage of the circular form is the fact that the cross head can adjust itself to bring the wrist pin parallel to the crank pin. The guides can be bored at the same setting as the cylinder in small engines and thus secure good alignment.

31. Stuffing Boxes. In steam engines and pumps the glands for holding the steam and water packing are the sliding bearings which cause the greatest friction and the most trouble. Fig. 21 shows the general arrangement. B is the stuffing box attached to the cylinder head; R is the piston rod; G the gland adjusted by nuts on the studs shown; P the packing contained in a recess in the box and consisting of rings, either of some elastic fibrous material like hemp and woven rubber cloth or of some soft metal like babbit. The pressure between the packing and the rod, necessary to prevent leakage of steam or water, is the cause of considerable friction and lost work. Experiments made from time to time in the laboratories of the Case School of Applied Science have shown the extent and manner of variation of this friction. The results for steam packings may be summarized as follows :

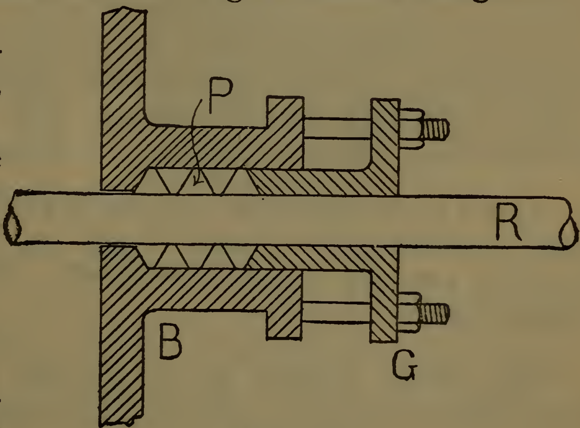


Fig. 21.

1. That the softer rubber and graphite packings,

which are self-adjusting and self-lubricating, as in Nos. 2, 3, 7, 8, and 11, consume less power than the harder varieties. No. 17, the old braided flax style, gave very good results.

2. That oiling the rod will reduce the friction with any packing.

3. That there is almost no limit to the loss caused by the injudicious use of the monkey-wrench.

4. That the power loss varies almost directly with the steam pressure in the harder varieties, while it is approximately constant with the softer kinds.

The diameter of rod used—two inches—would be appropriate for engines from 50 to 100 horse-power. The piston speed was about 140 feet per minute in the experiments, and the horse-power varied from .036 to .400 at 50 pounds steam pressure, with a safe average for the softer class of packings of .07 horse-power.

At a piston speed of 600 feet per minute, the same friction would give a loss of from .154 to 1.71 with a working average of .30 horse-power, at a mean steam pressure of 50 pounds.

In Table 12 Nos. 6, 14, 15 and 16 are square, hard rubber packings without lubricants.

Similar experiments on hydraulic packings under a water pressure varying from ten to eighty pounds per square inch gave results as shown in Table 14.

The figures given are for a two inch rod running at an overage piston speed of 140 feet per minute.

TABLE XII.

Kind of Packing.	No. Trials.	Total Time of Run in Minutes.	Av. Horse-Power Consumed by each Box.	Horse Pow. Cons. at 50 Lbs. Press.	Remarks on Leakage, etc.
1	5	22	.091	.085	Moderate leakage.
2	8	40	.049	.048	Easily adjusted; slight leakage.
3	5	25	.037	.036	Considerable leakage.
4	5	25	.159	.176	Leaked badly.
5	5	25	.095	.081	Oiling necessary; leaked badly.
6	5	25	.368	.400	Moderate leakage.
7	5	25	.067	.067	Easily adjusted and no l'kage.
8	5	25	.082	.082	Very satisfactory; slight l'kage.
9	3	15	.200	.182	Moderate leakage.
10	3	. .	.275	. .	Excessive leakage.
11	5	25	.157	.172	Moderate leakage.
12	5	25	.266	.330	Moderate leakage.
13	5	25	.162	.230	No leakage; oiling necessary.
14	5	25	.176	.276	Moderate l'kage; oiling neces.
15	5	25	.233	.255	Difficult to adjust; no leakage.
16	5	25	.292	.210	Oiling necessary; no leakage.
17	5	25	.128	.084	No leakage.

TABLE XIII.

Kind of Packing.	Horse Power consumed by each Box, when Pressure was applied to Gland Nuts by a Seven - Inch Wrench.						Horse Power before and after oiling Rod.	
	5 Pounds	8 Pounds	10 Pounds	12 Pounds	14 Pounds	16 Pounds	Dry.	Oiled.
1	.120	......	.136	......	......	......	......	......
3	......	......	......	......	......	......	.055	.021
4	......	.248	.....	.303	......	.390	.154	.123
5	......	.220	......	......	......	......	......	......
6	......	.348	.430	......	......	......	.323	.194
7	......	.126	.228	.260	.330	.340	.067	.053
8	......	.363	.500	.535	.520	.533	.533	.236
9	......	.666	......	......	......	......	.666	.636
11	......	.405	.454	......	......	......	.454	.176
12	......	.161	.242	.359	.454	......	.454	.122
13	......	.317	.394	.582	......	......	......	......
15	......	.526	......	......	......	......	......	......
16	......	.327	.860	......	......	......	......	.....
17	......	.198	.277	.380	......	......	......	......

TABLE XIV.

No. of Packing.	Av. H. P. at 20 Lbs.	Av. H. P. at 70 Lbs.	Max. H. P.	Min. H. P.	Av. H. P. for entire Test.
1	.077	.351	.452	.024	.259
2	.422	.500	.512	.167	.410
3	.130	.178	.276	.035	.120
4	.184	.195	.230	.142	.188
5	.146	.162	.285	.069	.158
6	.240	.200	.255	.071	.186
7	.127	.192	.213	.095	.154
8	.153	.174	.238	.112	.165
9	.287	.469	.535	.159	.389
10	.151	.160	.226	.035	.103
11	.141	156	.380	.064	.177
12	.053	.095	.143	.035	.090

Packings Nos. 5, 6, 10 and 12 are braided flax with graphite lubrication and are best adapted for low pressures. Packings Nos. 3, 4 and 7 are similar to the above but have parafine lubrication. Packings Nos. 2 and 9 are square duck without lubricant and are only suitable for very high pressures, the friction loss being approximately constant.

EXAMPLES.

Make a careful study and sketch of the sliding bearings on each of the following machines and analyze as to: (a) Purpose (b) Character. (c) Adjustment. (d) Lubrication.

1. One of the engine lathes in the shop.
2. One of the planing machines.
3. One of the shaping machines.
4. One of the milling machines.
5. One of the upright drills.
6. One of the engines.

Chapter 7.

JOURNALS, PIVOTS AND BEARINGS.

32. **Journals.** A journal is that part of a rotating shaft which rests in the bearings and is of necessity a surface of revolution, usually cylindrical or conical. The material of the journal is generally steel, sometimes soft and sometimes hardened and ground.

The material of the bearing should be softer than the journal and of such a quality as to hold oil readily. The cast metals such as cast iron, bronze and babbitt metal are suitable on account of their porous, granular character. Wood, having the grain normal to the bearing surface, is used where water is the lubricant, as in water wheel steps and stern bearings of propellers,

33. **Adjustment.** Bearings wear more or less rapidly with use and need to be adjusted to compensate for the wear. The adjustment must be of such a character and in such a direction as to take up the wear and at the same time maintain as far as possible the correct shape of the bearing. The adjustment should then be in the line of the greatest pressure.

Fig. 22 illustrates some of the more common ways of adjusting a bearing, the arrows showing the direction of adjustment and presumably the direction of pressure. (a) is the most usual where the principal wear is vertical. (d) is a form frequently used on the main journals of

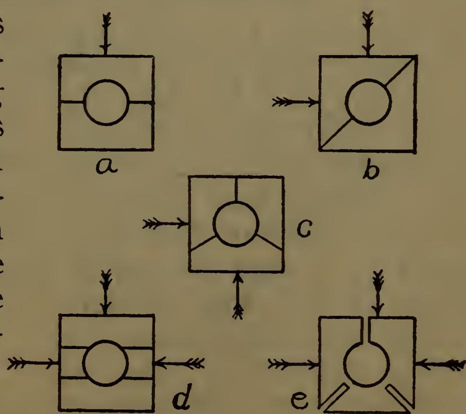


Fig. 22.

engines when the wear is in two directions, horizontal on account of the steam pressure and vertical on account of the weight of shaft and fly wheel. All of these are more or less imperfect since the bearing, after wear and adjustment, is no longer cylindrical but is made up of two or more approximately cylindrical surfaces.

A bearing slightly conical and adjusted endwise as it wears, is probably the closest approximation to correct practice.

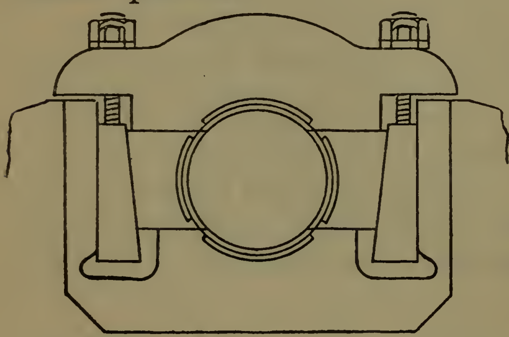


Fig. 23.

Fig. 23 shows the main bearing of the Porter - Allen engine, one of the best examples of a four part adjustment. The cap, is adjusted in the normal way with bolts and nuts; the bottom, can be raised and lowered by liners placed underneath; the cheeks can be moved in or out by means of the wedges shown. Thus it is possible, not only to adjust the bearing for wear, but to align the shaft perfectly.

The main bearing of the spindle in a lathe, as shown in Fig. 24, offers a good example of symmetrical adjustment. The headstock A has a conical hole to receive the bearing B, which latter can be moved lengthwise by the nuts F G. The bearing may be split into two, three or four segments or it may be cut as shown in (e) Fig. 22 and sprung into adjustment. A careful distinction must be made between this class of bearing and that before

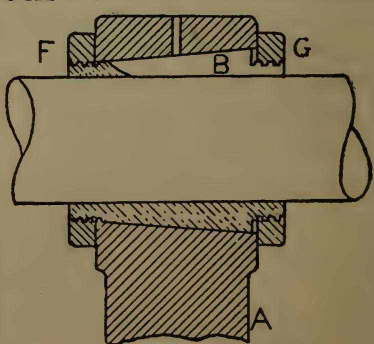


Fig. 24.

mentioned, where the journal itself is conical and adjusted endwise. A good example of the latter form is seen in the spindles of many milling machines.

34 **Lubrication.** The bearings of machines which run intermittently, like most machine tools, are oiled by means of simple oil holes, but machinery which is in continuous motion as is the case with line shafting and engines requires some automatic system of lubrication. There is not space in these notes for a detailed description of all the various types of oiling devices and only a general classification will be attempted.

Lubrication is effected in the following ways:

1. By grease cups.
2. By oil cups.
3. By oily pads of felt or waste.
4. By oil wells with rings or chains for lifting the oil.
5. By centrifugal force through a hole in the journal itself.

Grease cups have little to recommend them except as auxiliary safety devices. Oil cups are various in their shapes and methods of operation and constitute the chief class of lubricating devices. They may be divided according to their operation into wick oilers, needle feed, and sight feed. The two first mentioned are nearly obsolete and the sight feed oil cup, which drops the oil at regular intervals through a glass tube in plain sight, is in common use. The best sight feed oiler is that which can be readily adjusted as to time intervals, which can be turned on or off without disturbing the adjustment and which shows clearly by its appearance whether it is turned on. On engines and electric machinery which is in continuous use day and night, it is very important that the oiler itself should be stationary, so that it may be filled without stopping the machinery.

For continuous oiling of stationary bearings as in line shafting and electric machinery, an oil well below

the bearing is preferred, with some automatic means of pumping the oil over the bearing, when it runs back by gravity into the well. Porous wicks and pads acting by capillary attraction are uncertain in their action and liable to become clogged. For bearings of medium size, one or more light steel rings running loose on the shaft and dipping into the oil, as shown in Fig. 25, are the best. For large bearings flexible chains are employed which take up less room than the ring. Centrifugal oilers are most used on parts which cannot readily be oiled when in motion, such as loose pulleys and the crank pins of engines.

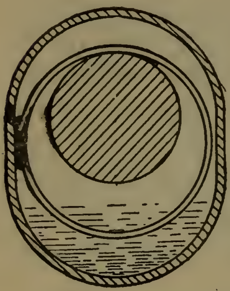


Fig. 25

Fig. 26 shows two such devices as applied to an engine. In A the oil is supplied by the waste from the main journal; in B an external sight feed oil cup is used which supplies oil to the central revolving cup C.

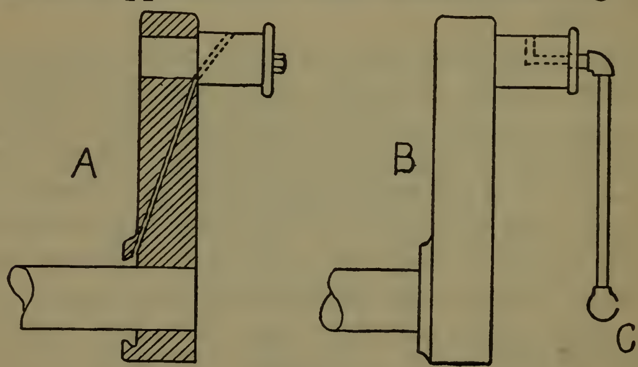


Fig. 26.

Loose pulleys or pulleys running on stationary studs are best oiled from a hole running along the axis of the shaft and thence out radially to the surface of the bearing. A loose bushing of some soft metal perforated with holes is a good safety device for loose pulleys.

Note: For adjustable pedestal and hanger bearings see the chapter on shafting.

35. Friction of Journals:

Let W = the total load on a journal in lbs.
l = the length of journal in inches.
d = the diameter of journal in inches.
N = number of revolutions per minute.
v = velocity of rubbing in feet per minute.
F = friction at surface of journal in lbs.
= W tan φ nearly.

If a journal is properly fitted in its bearing and does not bind, the value of F will not exceed W tan φ and may be slightly less. The value of tan φ varies according to the materials used and the kind of lubrication, from .05 to .01 or even less. The work absorbed in friction may be thus expressed:

$$Fv = W\tan\varphi \times \frac{\pi dN}{12} = \frac{\pi dNW\tan\varphi}{12} \qquad (56)$$

36. Limits of Pressure. Too great an intensity of pressure between the surface of a journal and its bearing will force out the lubricant and cause heating and possibly "siezing". The safe limit of pressure depends on the kind of lubricant, the manner of its application and upon whether the pressure is continuous or intermittent. The projected area of a journal, or the product of its length by its diameter, is used as a divisor.

The journals of railway cars offer a good example of continuous pressure and severe service. A limit of 300 lbs. per square inch of projected area has been generally adopted in such cases.

In the crank and wrist pins of engines, the reversal of pressure diminishes the chances of the lubricant being squeezed out, and a pressure of 500 lbs. per sq. in. is generally allowed.

The use of heavy oils or of an oil bath, and the employment of harder materials for the journal and its bearing allow of even greater pressures.

37. Heating of Journals. The proper length of journals depends on the liability of heating.

The energy or work expended in overcoming friction is converted into heat and must be conveyed away by the material of the rubbing surfaces. If the ratio of this energy to the area of the surface exceeds a certain limit, depending on circumstances, the heat will not be conveyed away with sufficient rapidity and the bearing will heat.

The area of the rubbing surface is proportional to the projected area or product of the length and diameter of the journal, and it is this latter area which is used in calculation.

Adopting the same notation as is used in Art. 35, we have from equation (56)

$$\text{the work of friction} = \frac{\pi \text{dNWtan}\varphi}{12} \text{. ft. lbs.}$$

$$\text{or} = \pi \text{dNWtan}\,\varphi \text{ inch lbs.}$$

The work per square inch of projected area is then:

$$\text{w} = \frac{\pi \text{dNWtan}\varphi}{\text{ld}} = \frac{\pi \text{NWtan}\varphi}{\text{l}} \quad \ldots\ldots\ldots\ldots \text{(a)}$$

Solving in (a) for l

$$\text{l} = \frac{\pi \text{NWtan}\varphi}{\text{w}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(b)}$$

Let $\frac{\text{w}}{\pi \tan\varphi} = \text{C}$ a co-efficient whose value is to be obtained by experiment; then

$$\text{C} = \frac{\text{WN}}{\text{l}} \text{ and } \text{l} = \frac{\text{WN}}{\text{C}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (57)$$

Crank pins of steam engines have perhaps caused more trouble by heating than any other form of journal. A comparison of eight different classes of propellers in the old U. S. Navy showed an average value for C of 350000.

A similar average for the crank pins of thirteen screw steamers in the French Navy gave C=400000.

Locomotive crank pins which are in rapid motion through the cool outside air allow a much larger value of C, sometimes more than a million.

Examination of ten modern stationary engines shows an average value of C=200000 and an average pressure per square inch of projected area=300 lbs.

In general we may use these formulas for stationary practice:

To prevent heating $l=\frac{WN}{200000}$ ....(58)

To prevent wear $l\,d=\frac{W}{300}$ ....(59)

38. **Strength and Stiffness of Journals.** A journal is usually in the condition of a bracket with a uniform load, and the bending moment $M=\frac{Wl}{2}$

Therefore by formula (6)

$$d=\sqrt[3]{\frac{10.2M}{S}}=\sqrt[3]{\frac{5.1Wl}{S}}$$

$$\text{or } d=1.721\sqrt[3]{\frac{Wl}{S}} \quad \text{..... ....(60)}$$

The maximum deflection of such a bracket is

$$\triangle=\frac{Wl^3}{8EI}$$

$$I=\frac{\pi d^4}{64}=\frac{Wl^3}{8E\triangle}$$

$$d^4=\frac{64Wl^3}{8\pi E\triangle}=\frac{2.547Wl^3}{E\triangle}$$

If as is usual $\triangle$ is allowed to be $\frac{1}{100}$ inches, then

for stiffness $d=\sqrt[4]{\frac{254.7Wl^3}{E}}$ ....(61)

or approximately $d=4\sqrt[4]{\frac{Wl^3}{E}}$ ....(62)

The designer must be guided by circumstances in determining whether the journal shall be calculated for wear, for strength or for stiffness. A safe way is to use all three of the formulas and take the largest result.

Remember that no factor of safety is needed in formula for stiffness.

Note that W in formulas for strength and stiffness is not the average but the maximum load.

39. **Caps and Bolts.** The cap of a journal bearing is in the condition of a beam supported by the holding down bolts and loaded at the center, and may be designed either for strength or for stiffness.

Let: P=max. upward pressure on cap.
L=distance between bolts.
b=breadth of cap at center.
h=depth of cap at center.
$\triangle$=greatest allowable deflection.

Strength: $M=\frac{Sbh^2}{6}=\frac{PL}{4}$

$$h=\sqrt{\frac{3PL}{2bS}} \quad \text{.......................} \quad (63)$$

Stiffness: $\triangle=\frac{WL^3}{48EI}$

$$I=\frac{bh^3}{12}=\frac{WL^3}{48E\triangle}$$

$$h=\sqrt[3]{\frac{WL^3}{4bE\triangle}} \quad \text{....................} \quad (64)$$

If $\triangle$ is allowed to be $\frac{1}{100}$ inches and E for cast iron is taken = 18000000.

then: $$h=.01115L\sqrt[3]{\frac{W}{b}} \quad \text{.......................} \quad (65)$$

The holding down bolts should be so designed that the bolts on one side of the cap may be capable of carrying safely two thirds of the total pressure.

EXAMPLES.

1. A flat car weighs 10 tons, is designed to carry a load of 20 tons more and is supported by two four wheeled trucks, the axle journals being of wrought iron and the wheels 33 inches in diameter.

Design the journals, considering heating, wear, strength and stiffness, assuming a maximum speed of 30 miles an hour, factor of safety$=10$ and $C=300000$.

2. Measure the crank pin of any modern engine which is accessible, calculate the various constants and compare them with those given in this section.

3. Design a crank pin for an engine under the following conditions:

Diameter of piston = 28 inches.

Maximum steam pressure = 90 lbs. per sq. in.
Mean steam pressure = 40 lbs. per sq. in.
Revolutions per minute = 75

Determine dimensions necessary to prevent wear and heating and then test for strength and stiffness.

4. Make a careful study and sketch of journals and journal bearings on each of the following machines and analyze as to (a) Materials, (b) Adjustment, (c) Lubrication.

(1) One of the engine lathes in the shop.
(2) One of the milling machines.
(3) One of the steam engines.
(4) One of the electric generators.

5. Sketch at least one form of oil cup used in the laboratories and explain its working.

6. The shaft journal of a vertical engine is 4 ins. in diameter by 6 ins. long. The cap is of cast iron, held down by 4 bolts of wrought iron, each 5 ins. from center of shaft, and the greatest vertical pressure is 12000 lbs.

Calculate depth of cap at center for both strength and stiffness, and also the diameter of bolts.

7. Investigate the strength of the cap and bolts of

some pillow block whose dimensions are known, under a pressure of 500 lbs. per sq. in. of projected area.

8. The total weight on the drivers of a locomotive is 64000 lbs. The drivers are four in number, 5 ft. 2 in. in diameter, and have journals $7\frac{1}{2}$ in. in diameter.

Determine the horse power consumed in friction under each of the three above named conditions, when the locomotive is running 50 miles an hour, assuming $\tan\varphi = .05$.

40. Friction of Pivots or Step-Bearings.— Flat Pivots.

Let W = weight on pivot
d_1 = outer diameter of pivot
p = intensity of vertical pressure
M = moment of friction
f = co-efficient of friction $= \tan\varphi$

We will assume p to be a constant which is no doubt approximately true.

$$\text{Then} \quad p = \frac{W}{\text{area}} = \frac{4W}{\pi d_1^2}$$

Let r = the radius of any elementary ring of a width = dr , then area of element $= 2\pi r dr$

Friction on element $= fp \times 2\pi r dr$

Moment of friction of element $= 2fp\pi r^2 dr$

$$\text{and} \qquad M = 2fp\pi \int_0^{\frac{d^1}{2}} r^2 dr \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(a)}$$

$$\text{or } M = 2fp\pi\frac{r^3}{3} = 2fp\pi\frac{d_1^3}{24}$$

$$= \frac{2f\pi d_1^3}{24} \times \frac{4W}{\pi d_1^2} = \frac{1}{3}Wfd_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(66)}$$

The great objection to this form of pivot is the uneven wear due to the difference in velocity between center and circumference.

41. Flat Collar.

Let $d_2 =$ inside diameter

Integrating as in equation (a) above, but using limits $\frac{d_1}{2}$ and $\frac{d_2}{2}$ we have

$$M = 2fp\pi\frac{d_1^3 - d_2^3}{24}$$

In this case

$$p = \frac{4W}{\pi(d_1^2 - d_2^2)}$$

and

$$M = \tfrac{1}{3}Wf\frac{d_1^3 - d_2^3}{d_1^2 - d_2^2} \qquad \text{.........................(67)}$$

42. Conical Pivot.

Let $\alpha =$ angle of inclination to the vertical.

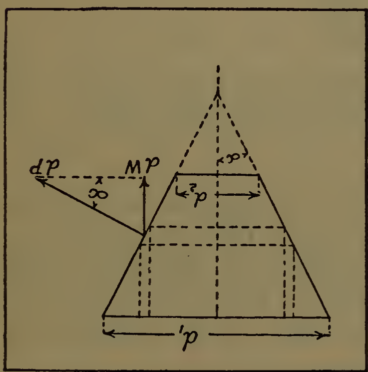


Fig. 27.

As in the case of a flat ring the intensity of the vertical pressure is

$$p = \frac{4W}{\pi(d_1^2 - d_2^2)}$$

and the vertical pressure on an elementary ring of the bearing surface is

$$dW = \frac{4W}{\pi(d_1^2 - d_2^2)} \times 2\pi\, rdr = \frac{8Wrdr}{d_1^2 - d_2^2}$$

As seen in Fig. 27 the normal pressure on the elementary ring is

$$dP = \frac{dW}{\sin\alpha} = \frac{8Wrdr}{(d_1^2 - d_2^2)\sin\alpha}$$

The friction on the ring is fdP and the moment of this friction is

$$dM = frdP = \frac{8Wfr^2dr}{(d_1^2 - d_2^2)\sin\alpha}$$

$$M = \frac{8Wf}{(d_1^2 - d_2^2)\sin\alpha}\int_{\frac{d_2}{2}}^{\frac{d_1}{2}} r^2dr$$

$$= \tfrac{1}{3}\frac{Wf}{\sin\alpha}\,\frac{d_1^3 - d_2^3}{d_1^2 - d_2^2} \quad \text{.........................} (68)$$

As α approaches $\frac{\pi}{2}$ the value of M approaches that of a flat ring, and as α approaches 0 the value of M approaches ∞.

If $d_2 = 0$ we have

$$M = \tfrac{1}{3}\frac{Wfd}{\sin\alpha} \quad \text{.........................} (69)$$

The conical pivot also wears unevenly, usually assuming a concave shape as seen in profile.

43. Schiele's Pivot. By experimenting with a pivot and bearing made of some friable material, it was shown that the outline tended to become curved as shown in Fig. 29. This led to a mathematical investigation which showed that the curve would be a tractrix under certain conditions.

This curve may be traced mechanically as shown in Fig. 28.

Let the weight W be free to move on a plane. Let the string SW be kept taut and the end S moved along the straight line SL. Then will a pencil attached to the center of W trace on the plane a tractrix whose axis is SL.

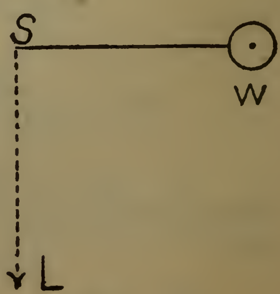


Fig. 28.

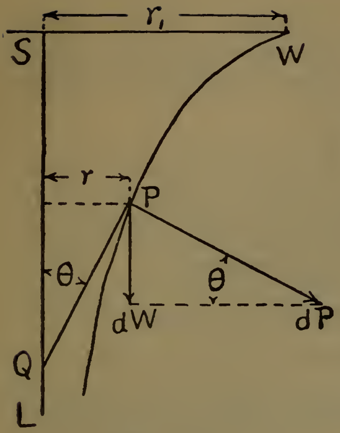


Fig. 29.

In Fig. 29 let SW=length of string = r_1 and let P be any point in the curve. Then it is evident that the tangent PQ to the curve is a constant and $=r_1$

Also $\frac{r}{\sin\theta}=r_1$

Let a pivot be generated by revolving the curve around its axis SL. As in the case of the conical pivot it can be proved that the normal pressure on an element of convex surface is

$$dP=\frac{8Wrdr}{(d_1^2-d_2^2)\sin\theta} \quad \text{(a)}$$

Let the normal wear of the pivot be assumed to be proportional to this normal pressure and to the velocity of the rubbing surfaces, *i. e.* normal wear proportional to pr, then is the vertical wear proportional to $\frac{pr}{\sin\theta}$. But $\frac{r}{\sin\theta}$ is a constant, therefore the vertical wear will be the same at all points. This is the characteristic feature and advantage of this form of pivot.

As shown in equation (a)

$$d\,P=\frac{8Wr_1dr}{d_1^2-d_2^2}$$

$$\therefore \; d\,M=\frac{8Wfr_1rdr}{d_1^2-d_2^2}$$

and

$$M=\frac{8Wfr_1}{d_1^2-d_2^2}\,\frac{r_1^2-r_2^2}{2}=\frac{Wfd_1}{2} \quad \text{(70)}$$

M is thus shown to be independent of d_2 or of the length of pivot used.

This pivot is sometimes wrongly called anti-friction. As will be seen by comparing equations (66) and (70) the moment of friction is fifty per cent. greater than that of the common flat pivot.

The distinct advantage of the Schiele pivot is in the fact that it maintains its shape as it wears and is self-adjusting. It is an expensive bearing to manufacture and is seldom used on that account.

It is not suitable for a bearing where most of the pressure is side ways.

44. **Multiple Bearings.** To guard against abrasion in flat pivots a series of rubbing surfaces which divide the wear is sometimes provided, Several flat discs placed beneath the pivot and turning indifferently, may be used. Sometimes the discs are made alternately of a hard and a soft material. Bronze, steel and raw hide are the more common materials.

Notice in this connection the button or washer at the outer end of the head spindle of an engine lathe and the loose collar on the main journal of a milling machine. Pivots are usually lubricated through a hole at the center of the bearing and it is desirable to have a pressure head on the oil to force it in.

The compound thrust bearing generally used for propeller shafts consists of a number of collars of the same size forged on the shafts at regular intervals and dividing the end thrust between them, thus reducing the intensity of pressure to a safe limit without making the collars unreasonably large.

A safe value for p the intensity of pressure is, according to Whitham, 60 lbs. per sq. in. for high speed engines.

A table given by Prof. Jones in his book on Machine Design shows the practice at the Newport News ship-yards on marine engines of from 250 to 5000 H. P. The outer diameter of collars is about one and one-half times the diameter of the shafts in each case and the number of collars used varies from 6 in

the smallest engine to 11 in the largest. The pressure per sq. in. of bearing surface varies from 18 to 46 lbs. with an average value of about 32 lbs.

EXAMPLES.

1. Design and draw to full size a Schiele pivot for a water wheel shaft 4 inches in diameter, the total length of the bearing being 3 inches.

Calculate the horse-power expended in friction if the total vertical pressure on the pivot is two tons and the wheel makes 150 revs. per min. and assuming f = .25 for metal on wet wood.

2. Design a compound thrust bearing for a propeller shaft the diameters being 14 and 21 inches, the total thrust being 80000 lbs. and the pressure 40 lbs. per sq. in.

Calculate the horse-power consumed in friction and compare with that developed if a single collar of same area had been used. Assume f=.05 and revs. per min. = 120.

Chapter V.

BALL AND ROLLER BEARINGS.

45. **General Principles.** The object of interposing a ball or roller between a journal and its bearing, is to substitute rolling for sliding friction and thus to reduce the resistance. This can be done only partially and by the observance of certain principles. In the first place it must be remembered that each ball can roll about but one axis at a time; that axis must be determined and the points of contact located accordingly.

Secondly, the pressure should be approximately normal to the surfaces at the points of contact.

Finally it must be understood, that on account of the contact surfaces being so minute, a comparatively slight pressure will cause distortion of the balls and an entire change in the conditions.

46. **Journal Bearings.** These may be either two, three or four point, so named from the number of points of contact of each ball.

The axis of the ball may be assumed as parallel or inclined to the axis of the journal and the points of contact arranged accordingly. The simplest form consists of a plain cylindrical journal running in a bearing of the same shape and having rings of balls interposed. The successive rings of balls should be separated by thin loose collars to keep them in place. These collars are a source of rubbing friction, and to do away with them the balls are sometimes run in grooves either in journal, bearing or both.

Fig. 30 shows a bearing of this type, there being three points of contact and the axis of ball being parallel to that of journal.

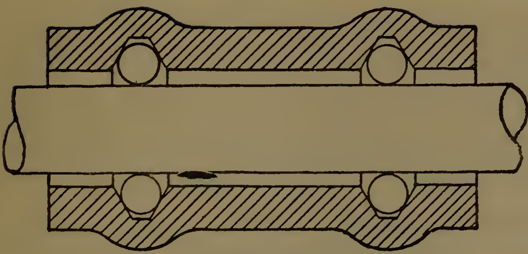


Fig. 30.

The bearings so far mentioned have no means of adjustment for wear. Conical bearings, or those in which the axes of the balls meet in a common point, supply this deficiency. In designing this class of bearings, either for side or end thrust, the inclination of the axis is assumed according to the obliquity desired and the points of contact are then so located that there shall be no slipping.

Fig. 31 illustrates a common form of adjustable or cone bearing and shows the method of designing a three point contact. A C is the axis of the cone, while the shaded area is a section of the cup, so called. Let a and b be two points of contact between ball and cup. Draw the line a b and produce to cut axis in A. Through the center of ball draw the line A B; then will this be the axis of rotation of the ball and a c, b d will be the projections of two circles of rotation. As the radii of theae circles have the same ratio as the radii of revolution a n, b m, there will be no slipping and the ball will roll as a cone inside another cone. The exact location of the third point of contact is not material. If it were at c, too much pressure would come on the cup at b; if at d there would be an excess of pressure at a but the rolling would be correct in either case. A convenient method is to locate p by drawing A D tangent to ball circle as shown. It is recommended however that the two opposing sur-

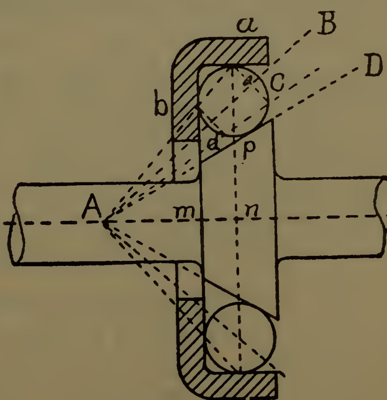


Fig. 31.

faces at p and b or a should make with each other an angle of not less than 25° to avoid sticking of the ball.

To convert the bearing just shown to four point contact, it would only be necessary to change the one cone into two cones tangent to the ball at c and d.

To reduce it to two point contact the points a and b are brought together to a point opposite p. As in this last case the ball would not be confined to a definite path it is customary to make one or both surfaces concave conoids with a radius about three fourths the diameter of the ball. See Fig. 32.

47. Step-Bearings. The same principles apply as in the preceding article and the axis and points of contact may be varied in the same way. The most common form of step-bearing consists of two flat circular plates separated by one or more rings of balls. Each ring must be kept in place by one or more loose retaining collars, and these in turn are the cause of some sliding friction. This is a bearing with two point contact and the balls turning on horizontal axes. If the space between the plates is filled with loose balls, as is sometimes done, the rubbing of the balls against each other will cause considerable friction.

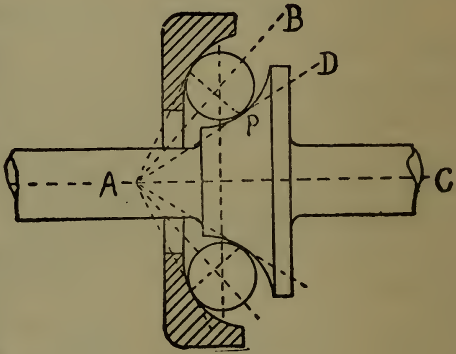


Fig. 32.

To guide the balls without rubbing friction three point contact is generally used.

Fig. 33 illustrates a bearing of this character. The method of design is shown in the figure the principle being the same as in Fig. 31. By comparing the lettering of the two figures the similarity will be readily seen.

This last bearing may be converted to four point contact by making the upper collar of the same shape as the lower. To guide the balls in two point contact use is sometimes made of a cage ring, a flat collar drilled with holes just a trifle larger than the balls and disposing them either in spirals or in irregular order.

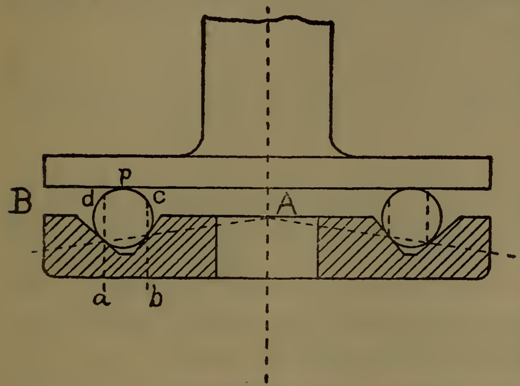


Fig. 33.

This method has the advantage of making each ball move in a path of different radius thus securing more even wear for the plates.

48. Materials and Wear. The balls themselves are always made of steel, hardened in oil, tempered and ground. They are usually accurate to within one ten thousandth of an inch. The plates, rings and journals must be hardened and ground in the same way and perhaps are more likely to wear out or fail than the balls. A long series of experiments made at the Case School of Applied Science on the friction and endurance of ball step-bearings showed some interesting peculiarities.

Using flat plates with one circle of quarter inch balls it was found that the balls pressed outward on the retaining ring with such force as to cut and indent it seriously. This was probably due to the fact that the pressure slightly distorted the balls and changed each sphere into a partial cylinder at the touching points. While of this shape it would tend to roll in a straight line or a tangent to the circle. Grinding the plates slightly convex at an angle of one to one and-a-half degrees obviated the difficulty to a certain extent. Under even moderately heavy loads

the continued rolling of the ring of balls in one path soon damaged the plates to such an extent as to ruin the bearing.

A flat bearing filled with loose balls developed three or four times the friction of the single ring and a three point bearing similar to that in Fig. 33 showed more than twice the friction of the two point.

A flat ring cage such as has already been described was the most satisfactory as regards friction and endurance.

The general conclusions derived from the experiments were that under comparatively light pressures the balls are distorted sufficiently to seriously disturb the manner of rolling and that it is the elasticity and not the compressive strength of the balls which must be considered in designing bearings.

49. **Design of Bearings.** Figures on the direct crushing strength of steel balls have little value for the designer. For instance it has been proved by numerous tests that the average crushing strengths of ¼ inch and ⅜ inch balls are about 7500 lbs. and 15000 lbs. respectively. Experiments made by the writer show that a ¼ inch ball loses all value as a transmission element on account of distortion, at any load of more than 100 lbs.

Prof. Gray states, as a conclusion from some experiments made by him, that not more than 40 lbs. per ball should be allowed for ⅜ inch balls.

This distortion doubtless accounts for the failure of theoretically correct bearings to behave as was expected of them. Ball bearings should be designed as has been explained in the preceding articles and then only used for light loads.

50. **Roller Bearings.** The principal disadvantage of ball bearings lies in the fact that contact is only at a point and that even moderate pressure causes excessive distortion and wear. The substitution of cylinders or cones for the balls is intended to overcome this difficulty.

The simplest form of roller bearing consists of a plain cylindrical journal and bearing with small cylindrical rollers interposed instead of balls. There are two difficulties here to be overcome. The rollers tend to work endways and rub or score whatever retains them. They also tend to twist around and become unevenly worn or even bent and broken, unless held in place by some sort of cage. In short they will not work properly unless guided and any form of guide entails sliding friction. The cage generally used is a cylindrical sleeve having longitudinal slots which hold the rollers loosely and prevent their getting out of place either sideways or endways.

The use of balls between the rollers at the ends has been tried with some degree of success. Large rollers have been turned smaller at the ends and the bearings then formed allowed to turn in holes bored in revolving collars. These collars must be so fastened or geared together as to turn in unison.

51. Hyatt Rollers. The tendency of the rollers to get out of alignment has been already noticed. The Hyatt roller is intended by its flexibility to secure uniform pressure and wear under such conditions. It consists of a flat strip of steel wound spirally about a mandrel so as to form a continuous hollow cylinder. It is true in form and comparatively rigid against compression, but possesses sufficient flexibility to adapt itself to slight changes of bearing surface. This bearing is readily caged by running rods through the rollers and riveting them to collars at the ends.

Experiments made by the Franklin Institute show that the Hyatt roller possesses a great advantage in efficiency over the solid roller.

Testing ¾ inch rollers between flat plates under loads increasing to 550 lbs. per linear inch of roller developed co-efficients of friction for the Hyatt roller from 23 to 51 per cent. less than for the solid roller.

Subsequent examination of the plates showed also a much more even distribution of pressure for the former.

52. Roller Step Bearings. In article 48 attention was called to the fact that the balls in a step-bearing under moderately heavy pressures tend to become cylinders or cones and to roll accordingly. This has suggested the use of small cones in place of the balls, rolling between plates one or both of which is also conical. A successful bearing of this kind with short cylinders in place of cones is used by the Sprague-Pratt Elevator Co., and is described in the American Machinist for June 27, 1901. The rollers are arranged in two spiral rows so as to distribute the wear evenly over the plates and are held loosely in a flat ring cage. This bearing has run well in practice under loads double those allowable for ball bearings, or over 100 lbs. per roll for rolls one-half inch in diameter and one-quarter inch long.

Chapter 9.

SHAFTING, COUPLINGS AND HANGERS.

53. Strength of Shafting.

Let D = diameter of the driving pulley or gear.
N = number revs. per minute.
P = force applied at rim.
T = twisting moment.

The distance through which P acts in one minute is πDN inches and work $= P\pi DN$ in. lbs. per min.

But $\frac{PD}{2} = T$ the moment, and $2\pi N =$ the angular velocity.

$\therefore$ work = moment $\times$ angular velocity

One horse power = 33000 ft. lbs. per min.
= 396000 in. lbs. per min.

$$\therefore HP = \frac{P\pi DN}{396000} = \frac{2\pi TN}{396000}$$

or

$$HP = \frac{TN}{63025} \qquad (71)$$

also

$$T = \frac{63025\ HP}{N} \qquad (72)$$

$$P = \frac{126050\ HP}{DN} \qquad (73)$$

The general formula for a circular shaft exposed to torsion alone is

$$d = \sqrt[3]{\frac{5.1\ T}{S}}$$

But

$$T = \frac{63025\ HP}{N} \text{ by (72)}$$

where N = no. revs. per min.

Substituting in formula for d

$$d = \sqrt[3]{\frac{321000\ HP}{SN}} \text{ nearly} \quad \ldots\ldots\ldots\ldots (74)$$

S may be given the following values:

45000 for common turned shafting.
50000 for rolled iron or soft steel. (0.15 C)
65000 for machinery steel. (0.55 C)

It is customary to use factors of safety for shafting as follows:

	f
Headshafts or prime movers	15
Line shafting	10
Short counters	6

The large factor of safety for head shafts is used not only on account of the severe service to which such shafts are exposed, but also on account of the inconvenience and expense attendant on failure of so important a part of the machinery. The factor of safety for line shafting is supposed to be large enough to allow for the transverse stresses produced by weight of pulleys, pull of belts, etc., since it is impracticable to calculate these accurately in most cases.

Substituting the values of S and introducing factors of safety, we have the following formulas for the safe diameters of the various kinds of shafts.

TABLE XV.

Kind of Shaft.	Material.		
	Common Iron	Soft Steel	Machy. Steel
Head Shaft.	$4.75 \sqrt[3]{\frac{HP}{N}}$	$4.58 \sqrt[3]{\frac{HP}{N}}$	$4.20 \sqrt[3]{\frac{HP}{N}}$
Line Shaft.	$4.15 \sqrt[3]{\frac{HP}{N}}$	$4.00 \sqrt[3]{\frac{HP}{N}}$	$3.67 \sqrt[3]{\frac{HP}{N}}$
Counter Shaft.	$3.50 \sqrt[3]{\frac{HP}{N}}$	$3.38 \sqrt[3]{\frac{HP}{N}}$	$3.10 \sqrt[3]{\frac{HP}{N}}$

In case there is a known bending moment M, combined with a known twisting moment T, then a resultant twisting moment

$$T' = M + \sqrt{M^2 + T^2}$$

is to be substituted for T in the above formulas.

54. Couplings. The flange or plate coupling is most commonly used for fastening together adjacent lengths of shafting.

Fig. 34 shows the proportions of such a coupling. The flanges are turned accurately on all sides, are keyed to the shafts and the two are centered by the projection of the shaft from one part into the other as shown at A. The bolts are turned to fit the holes loosely so as not to interfere with the alignment.

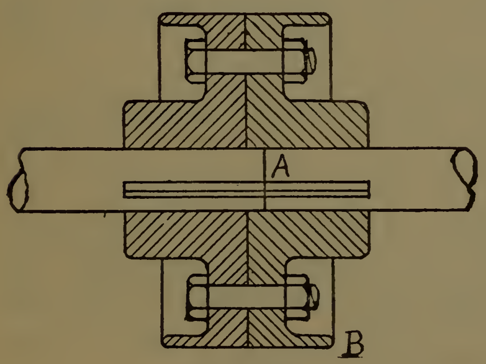


Fig. 34.

The projecting rim as at B prevents danger from belts catching on the heads and nuts of the bolts.

The faces of this coupling should be trued up in a lathe after being keyed to the shaft.

The sleeve coupling is less clumsy than the foregoing but is rather more complicated and expensive.

In Fig. 35 is illustrated a neat and effective coupling of this type. It consists of the sleeve S bored with two tapers and two threaded ends as shown. The two conical, split bushings B B are prevented from turning by the feather key K and are forced into the conical recesses by the two threaded collars C C and thereby clamped firmly to the shaft. The key K also nicks slightly the center of the main sleeve S, thus locking the whole combination.

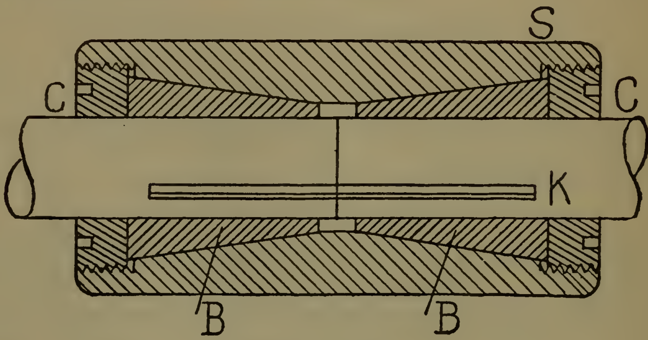


Fig. 35.

Couplings similar to this have been in use in the Union Steel Screw Works, Cleveland, Ohio, for many years and have given good satisfaction.

In another form of sleeve coupling the sleeve is split and clamped to the shaft by bolts passing through the two halves.

55. **Coupling Bolts.** The bolts used in the ordinary flange couplings are exposed to shearing, and their combined shearing moment should equal the twisting moment on the shaft.

Let n=number of bolts.
d_1=diameter of bolt.
D=diameter of bolt circle.

We will assume that the bolt has the same shearing strength as the shaft. The combined shearing strength of the bolt is $.7854d_1^2nS$ and their moment of resistance to shearing is

$$.7854d_1^2nS \times \frac{D}{2} = .3927Dd_1^2nS$$

This last should equal the torsion moment of the shaft or $.3927 D d_1^2 n S = \frac{Sd^3}{5.1}$

Solving for d_1 and assuming $D = 3d$ as an average value we have $d_1 = \frac{d}{\sqrt{6n}}$ ....(75)

In practice rather larger values are used than would be given by the formula.

56. Shafting Keys. The moment of the shearing stress on a key must also equal the twisting moment of the shaft.

Let b = breadth of a key.
l = length of key.
h = total depth of key.
S' = shearing strength of key.

The moment of shearing stress on key is

$$blS' \times \frac{d}{2} = \frac{bdlS'}{2}$$

and this must equal $\frac{Sd^3}{5.1}$ Usually $b = \frac{d}{4}$

For shafts of machine steel $S = S'$, and for iron shafts $S = \frac{3}{4}S'$ nearly as keys should always be of steel.

Substituting these values and reducing:

For iron shafting $l = 1.2d$ nearly.

For steel shafting $l = 1.6d$ nearly, as the least lengths of key to prevent its failing by shear.

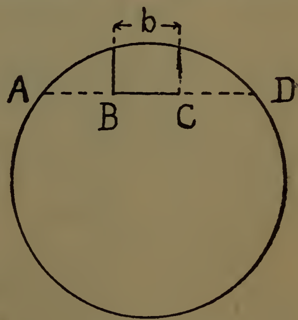


Fig. 35a.

If the key way is to be designed for uniform strength, the shearing area of the shaft on the line AB Fig. 35a should equal the shearing area of the key, if shaft and key are of the same material and $AB = CD = b$.

These proportions will make the depth of key way in shaft about $= \frac{5}{8}b$ and would be appropriate for a square key.

To avoid such a depth of key way which might weaken the shaft, it is better to use keys longer than required by preceding formulas. In American practice the total depth of key rarely exceeds $\frac{5}{8}b$ and one-half of this depth is in shaft.

To prevent crushing of the key the moment of the compressive strength of half the depth of key must equal T.

or $$\frac{d}{2} \times \frac{lh}{2} \times S_c = \frac{Sd^3}{5.1} \text{............(a)}$$

where S_c is the compressive strength of the key.

For iron shafts $S_c = 2S$

and for steel shafts $S_c = \frac{3}{2} S$

Substituting values of S_c and assuming $h = \frac{5}{8}b = \frac{5}{32} d$ we have

Iron shafts $l = 2.5d$ nearly

Steel shafts $l = 3\frac{1}{3}d$ nearly, as the least length for flat keys to prevent lateral crushing.

57. **Hangers and Boxes.** As shafting is usually hung to the ceiling and walls of buildings it is necessary to provide means for adjusting and aligning the bearings as the movement of the building disturbs them. Furthermore as line shafting is continuous and is not perfectly true and straight, the bearings should be to a certain extent self-adjusting. Reliable experiments have shown that usually one-half of the power developed by an engine is lost in the friction of shafting and belts. It is important that this loss be prevented as far as possible.

The boxes are in two parts and may be of bored cast iron or lined with Babbitt metal. They are usually about four diameters of the shaft in length and are oiled by means of a well and rings or wicks.

(See Chapter 7.) The best method of supporting the box in the hanger is by the ball and socket joint; all other contrivances such as set screws are but poor substitutes.

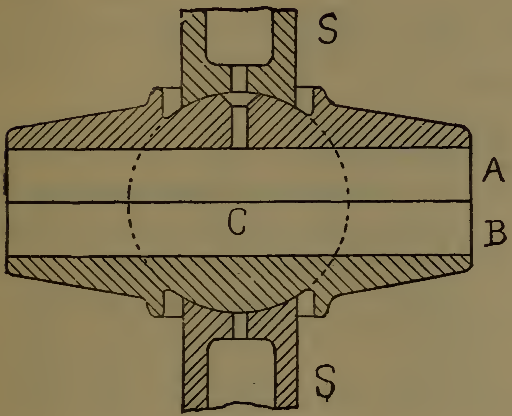


Fig. 36.

Fig. 36 shows the usual arrangement of the ball and socket.

A and B are the two parts of the box. The center is cast in the shape of a partial sphere with C as a center as shown by the dotted lines. The two sockets S S can be adjusted vertically in the hanger by means of screws and lock nuts. The horizontal adjustment of the hanger is usually effected by moving it bodily on the support, the bolt holes being slotted for this purpose.

Counter shafts are short and light and are not subject to much bending. Consequently there is not the same need of adjustment as in line shafting.

In Fig. 37 is illustrated a simple bearing for counters. The solid cast iron box B with a spherical center is fitted directly in a socket in the hanger H and held in position by the cap C and a set screw.

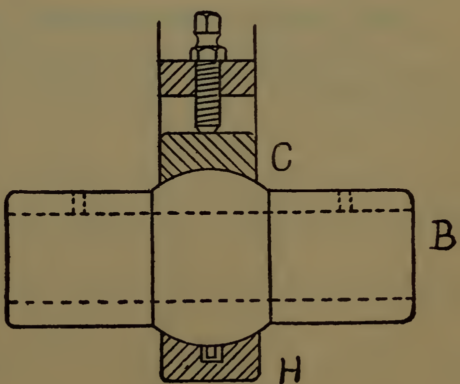


Fig. 37.

There is not space here to show the various forms of hangers and floor stands and reference is made to the catalogues of manufacturers. Hangers should be symmetrical, i. e. the center of the box should be in a vertical line with center of base. They should have relatively broad bases and should have the metal disposed to secure the greatest rigidity possible. Cored sections are to be preferred.

EXAMPLES.

1. Calculate the safe diameters of head shaft and three line shafts for a factory, the material to be rolled iron and the speeds and horse powers as follows:

Head shaft	100 HP	200 revs. per min.
Machine shop	30 HP	120 revs. per min.
Pattern shop	50 HP	250 revs. per min.
Forge shop	20 HP	200 revs. per min.

2. Determine the horse power of at least two lines of shafting whose speed and diameter are known.

3. Design and sketch to scale a flange coupling for a three inch line shaft including bolts and keys.

4. Design a sleeve coupling for the foregoing, different in principle from the one shown in the text.

5. Select the line shaft hanger which you prefer among those in the laboratories and make sketch and description of the same.

6. Do. for a countershaft hanger.

7. Explain in what way a floor stand differs from a hanger.

Chapter 10.

GEARS, PULLEYS AND FLY WHEELS.

58. Gear Teeth. The teeth of gears are made in three ways and are accordingly known as pattern molded, machine molded and machine cut, the first being the least accurate in form and the last the most so.

Let circular pitch $= p$

diameter pitch $= \frac{p}{\pi} = d$

pitch number $= \frac{\pi}{p} = \frac{1}{d}$

addendum $= a$

flank $= f$

clearance $= f - a = c$

height $= f + a = h$

width $= w$

space $= p - w = s$

The following table gives the ordinary proportions in use for the three kinds of teeth.

TABLE XVI. — PROPORTIONS OF GEAR TEETH.

Kind of Tooth.	Addendum. a	Flank. f	Clearance c	Height. h	Width. w	Space. s
Pattern Molded	.32 p	.38 p	.06 p	.7 p	.47 p	.53 p
Machine Molded	.32 p	.38 p	.06 p	.7 p	.48 p	.52 p
Machine Cut	d	1⅛d	⅛d	2⅛d	.5 p	.5 p

In making machine cut teeth some mechanics prefer to make the clearance $= \frac{1}{20}$ p which is slightly more than ⅛d.

The diameter pitch bears the same relation to the diameter as the circular pitch does to the circumference, and may be obtained by dividing the diameter by the number of teeth.

Its value is evidently $d = \frac{p}{\pi} = .3183\,p.$

The pitch number is simply the reciprocal of d and shows the number of teeth in the wheel per inch of diameter.

In modern practice the pitch number is either a whole number or a common fraction.

The following table gives values of the diameter and circular pitches for all the pitch numbers in use.

TABLE XVII.—PITCH OF GEAR TEETH.

Pitch Number. $\frac{1}{d}$	Diameter Pitch d	Circular Pitch p	Pitch Number. $\frac{1}{d}$	Diameter Pitch d	Circular Pitch p	Pitch Number. $\frac{1}{d}$	Diameter Pitch d	Circular Pitch p
½	2.	6.2832	7	.1429	.4488	24	.0417	.1309
¾	1.3333	4.1888	8	.125	.3927	26	.0385	.1208
1	1.	3.1416	9	.1111	.3491	28	.0357	.1122
1¼	.8	2.5133	10	.1	.3142	30	.0333	.1047
1½	.6667	2.0944	11	.0909	.2856	32	.0312	.0982
1¾	.5714	1.7952	12	.0833	.2618	34	.0294	.0924
2	.5	1.5708	13	.0769	.2417	36	.0278	.0873
2¼	.4444	1.3963	14	.0714	.2244	38	.0263	.0827
2½	.4	1.2566	15	.0667	.2094	40	.025	.0785
2¾	.3366	1.1424	16	.0625	.1963	42	.0238	.0748
3	.3333	1.0472	17	.0588	.1848	44	.0227	.0714
3½	.2857	.8976	18	.0555	.1745	46	.0217	.0683
4	.25	.7854	19	.0526	.1653	48	.0208	.0654
5	.2	.6283	20	.05	.1571	50	.02	.0628
6	.1667	.5236	22	.0455	.1428	56	.0179	.0561

The proportions given in Table XVI. are those most usual in practice, but many good authorities recommend a shorter tooth as giving less obliquity and sliding friction and as being much stronger.

The length generally recommended in such cases is equal to one-half the circular pitch plus the clearance. Short teeth are being more used every year.

59. Strength of Teeth.

Let P = total driving pressure on wheel at pitch circle. This may be distributed over two or more teeth, but the chances are against an even distribution.

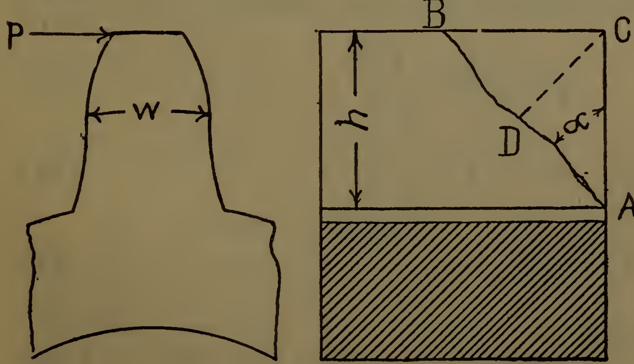
Again, in designing a set of gears the contact is likely to be confined to one pair of teeth in the smaller pinions.

Each tooth should therefore be made strong enough to sustain the whole pressure.

Rough Teeth. The teeth of pattern molded gears are apt to be more or less irregular in shape, and are especially liable to be thicker at one end on account of the draft of the pattern.

In this case the entire pressure may come on the outer corner of a tooth and tend to cause a diagonal fracture.

Let C in Fig. 39 be the point of application of the pressure P, and AB the line of probable fracture.



Drop the ⊥ CD on AB

Let AB = x and CD = y angle CAD = α

Fig. 39.

Use the notation of Fig. 38 and the proportions for pattern molded teeth in Table XVI.

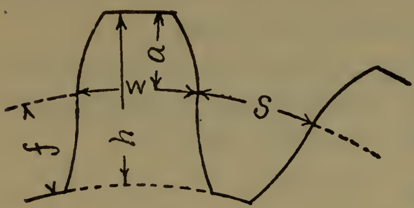


Fig. 38.

The bending moment at section AB is $M = Py$, and the moment of resistance is $M' = \frac{1}{6}Sxw^2$ where S = safe transverse strength of material.

$$Py = \frac{1}{6}Sxw^2$$

and $$S = \frac{6Py}{w^2x} \qquad \text{(a)}$$

If P and w are constant, then S is a maximum when $\frac{y}{x}$ is a maximum.

But $y = h \sin \alpha$ and $x = \frac{h}{\cos\alpha}$

$\frac{y}{x} = \sin\alpha \cos\alpha$ which is a maximum when $\alpha = 45°$ and $\frac{y}{x} = \frac{1}{2}$

Substituting this value in (a) we have $S = \frac{3P}{w^2}$

But in this case $w = .47p$ and therefore $S = \frac{3P}{.221p^2}$

and $$p = 3.684\sqrt{\frac{P}{S}} \qquad (76)$$

Diameter pitch, $$d = 1.173\sqrt{\frac{P}{S}} \qquad (77)$$

Pitch number, $$\frac{1}{d} = .853\sqrt{\frac{S}{P}} \qquad (78)$$

Unless machine molded teeth are very carefully made, it may be necessary to apply this rule to them as well.

Cut Gears. With careful workmanship machine molded and machine cut teeth should touch along the whole breadth. In such cases we may assume a line of contact at crest of tooth and a maximum bending moment

$$M = Ph$$

The moment of resistance at base of tooth is

$$M^1 = \tfrac{1}{6} Sbw^2$$

when b is the breadth of tooth.

In most teeth the thickness at base is greater than w, but in radial teeth it is less. Assuming standard proportions for cut gears:

$$h = 2\tfrac{1}{8}d = .6765p$$
$$w = .5p$$

and substituting above:

$$.6765\, Pp = \frac{Sbp^2}{24}$$

$$P = .0616bSp \quad \text{.........................(79)}$$

The above formula is general whatever the ratio of breath to pitch. The general practice in this country is to make $b = 3p$

Substituting this value of b in (79) and reducing:

$$p = 2.326\sqrt{\frac{P}{S}} \quad \text{..................... (80)}$$

or about two thirds the value obtained in Case I.

Diameter pitch $\quad d = .74\sqrt{\frac{P}{S}}$ .... (81)

Pitch number $\quad \frac{1}{d} = 1.35\sqrt{\frac{S}{P}}$ .... (82)

60. Lewis' Formulas. The foregoing formulas can only be regarded as approximate, since the strength of gear teeth depends upon the number of teeth in the wheel; the teeth of a rack are broader at the base and consequently stronger than those of a pinion. This is more particularly true of epicycloidal teeth.

Mr. Wilfred Lewis has deduced formulas which take into account this variation. For cut spur gears of standard dimensions the Lewis formula is as follows :

$$P = bSp\left(.124 - \frac{.888}{n}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (83)$$

where n = number of teeth.

This formula reduces to the same as (79) for n = 14 nearly.

Formula (79) would then properly apply only to small pinions, but as it would err on the safe side for larger wheels, it can be used where great accuracy is not needed. The same criticism applies to formulas (80) (81) and (82).

The value of S used should depend on the material and on the speed.

The following values are recommended for cast iron and cast steel.

Linear Velocity ft. per minute	0	500	1000	1500	2000
Cast Iron	6000	4500	2500	2000	1800
Cast Steel	15000	10000	7000	5000	4500

Good bronze will have about the same strength as the steel. The smaller values of S at the higher speeds are to allow for the blows and shocks which always occur in quick running gears.

61 Experimental Data. In the *American Machinist* for Jan. 14, 1897 are given the actual breaking loads of gear teeth which failed in service. The teeth had an average pitch of about 5 inches a breadth of about 18 inches and the rather unusual velocity of over 2000 ft. per minute. The average breaking load was about 15000 lbs. there being an average of about 50 teeth on the pinions. Substituting these values (83) and solving we get

$$S = 1575 \text{ lbs.}$$

This very low value is to be attributed to the condition of pressure on one corner noted in Art. 59. Substituting in formula for such a case.

$$S=\frac{3P}{.221p^2}=8150$$

This all goes to show that it is well to allow large factors of safety for rough gears, especially when the speed is high.

Experiments have been made ou the static strength of rough cast iron gear teeth at the Case School of Applied Science by breaking them in a testing machine. The teeth were cast singly from patterns, were two pitch and about 6 inches broad. The patterns were constructed accurately from templates representing 15° involute teeth and cycloidal teeth constructed with describing circle one-half the pitch circle of 15 teeth; the proportions used were those given for standard cut gears.

There were in all 41 cycloidal teeth of shapes corresponding to wheels of 15–24–36–48–72–120 teeth and a rack. There were 28 involute teeth corresponding to numbers above given omitting the pinion of 15 teeth.

The pressure was applied by a steel plunger tangent to the surface of tooth and so pivoted as to bear evenly across the whole breadth. The teeth were inclined at various angles so as to vary the obliquity from 0 to 25° for the cycloidal and from 15° to 25° for the involute. The point of application changing accordingly from the pitch line to the crest of the tooth. From these experiments the following conclusions were drawn;

1. The plane of fracture is approximately parallel to line of pressure and not necessarily at right angles to radial line, through center of tooth.

2. Corner breaks are likely to occur even when the pressure is apparently uniform along the tooth. There were fourteen such breaks in all.

3. With teeth of dimensions given, the breaking pressure per tooth varies from 25000 lbs. to 50000 lbs. for cycloids as the number of teeth increases from 15 to infinity; the breaking pressure for involutes of the same pitch varies from 34000 lbs to 80000 lbs. as the tooth number increases from 24 to infinity.

4. With teeth as above the average breaking pressure varies from 50000 lbs. to 26000 lbs. in the cycloids as the angle changes from 0° to 25° and the tangent point moves from pitch line to crest, with involute teeth the range is between 64000 and 39000 lbs.

5. Reasoning from the figures just given, rack teeth are about twice as strong as pinion teeth and involute teeth have an advantage in strength over cycloidal of from forty to fifty per cent. The advantage of short teeth in point of strength can also be seen. The modulus of rupture of the material used was about 36000 lbs. Values of S calculated from Lewis' formula for the various tooth numbers are quite uniform and average about 40000 lbs. for cycloidal teeth. Involute teeth are to-day generally preferred by manufacturers. William Sellers & Co. use an obliquity of 20° instead of 14½ or 15° the usual angle.

62. Teeth of Bevel Gears. There have been many formulas and diagrams proposed for determining the strength of bevel gear teeth, some of them being very complicated and inconvenient. It will usually answer every purpose from a practical standpoint, if we treat the section at the middle of the breadth of such a tooth as a spur wheel tooth and design it by the foregoing formulas. The breadth of the teeth of a bevel gear should be about one-third of the distance from the base of the cone to the apex.

One point needs to be noted; the teeth of bevel gears are stronger than those of spur gears of the same pitch and number of teeth since they are developed from a pitch circle having an element of the normal cone as a radius. To illustrate we will suppose that we are designing the teeth of a miter gear and

that the number of teeth is 32. In such a gear the element of normal cone is $\sqrt{2}$ times the radius. The actual shape of the teeth will then correspond to those of a spur gear having $32\sqrt{2} = 45$ teeth nearly.

Note.—In designing the teeth of gears where the number is unknown, the approximate dimensions may first be obtained by formula (80) and then these values corrected by using Lewis' formula.

EXAMPLES.

1. The drum of a hoist is 8 ins. in diameter and makes 5 revs. per minute. The diameter of gear on the drum is 36 inches and of its pinion 6 ins. The gear on the counter shaft is 24 ins. in diameter and its pinion is 6 ins. in diameter. The gears are all rough.

Calculate the pitch and number of teeth of each gear, assuming a load of one ton on drum chain and $S = 6000$. Also determine the horse-power of the machine.

2. Calculate the pitch and number of teeth of a cut cast steel gear 10 ins. in diameter, running at 250 revs. per min. and transmitting 20 HP.

3. A cast iron gear wheel is 30 ft. 6⅔ ins. in pitch diameter and has 192 teeth, which are machine cut and 30 ins. broad.

Determine the circular and diameter pitches of the teeth and the horse-power which the gear will transmit safety when making 12 revs. per min.

4. A two pitch cycloidal tooth, 6 ins. broad, 72 teeth to the wheel, failed under a load of 38000 lbs. Find value of S by Lewis' formula.

5. A vertical water wheel shaft is connected to horizontal head shaft by cast iron gears and transmits 150 H P. The water wheel makes 200 revs. per min. and the head shaft 100.

Determine the dimensions of the gears and teeth if the latter are approximately two pitch.

63. Rim and Arms. The rim of a gear, especially if the teeth are cast, should have nearly the same thickness as the base of tooth, to avoid cooling strains.

It is difficult to calculate exactly the stresses on the arms of the gear, since we know so little of the initial stress present, due to cooling and contraction. A hub of unusual weight is liable to contract in cooling after the arms have become rigid and cause severe tension or even fracture at the junction of arm and hub.

A heavy rim on the contrary may compress the arms so as actually to spring them out of shape. Of course both of these errors should be avoided, and the pattern be so designed that cooling shall be simultaneous in all parts of the casting.

The arms of spur gears are usually made straight without curves or taper, and of a flat, elliptical cross-section, which offers little resistance to the air. To support the wide rims of bevel gears and to facilitate drawing the pattern from the sand, the arms are sometimes of a rectangular or T section, having the greatest depth in the direction of the axis of the gear. For pulleys which are to run at a high speed it is important that there should be no ribs or projections on arms or rim which will offer resistance to the air. Experiments by the writer have shown this resistance to be serious at speeds frequently used in practice.

A series of experiments conducted by the author are reported in the *American Machinist* for Sep. 22, 1898, to which paper reference is here made.

Twenty-four pulleys having 3½ inches face and diameters of 16, 20 and 24 inches were broken in a testing machine by the pull of a steel belt, the ratio of the belt tension being adjusted by levers so as to be two to one. Twelve of the pulleys were of the ordinary cast iron type having each six arms tapering and of an elliptic section. The other twelve were Medart pulleys with steel rims riveted to arms and having some six and some eight arms. Test pieces cast from

the same iron as the pulleys showed an average modulus of rupture of 35800 for the cast iron and 50800 for the Medart.

In every case the arm or the two arms nearest the side of the belt having the greatest tension, broke first showing that the torque was not evenly distributed by the rim. Measurements of the deflection of the arms showed it to be from two to six times as great on this side as on the other. The buckling and springing of the rim was very noticeable especially in the Medart pulleys.

The arms of all the pulleys broke at the hub showing the greatest bending moment there as the strength of the arms at the hub was about double that at the rim. On the other hand some of the cast iron arms broke simultaneously at hub and rim showing a negative bending moment at the rim about one-halt that at the hub.

The following general conclusions are justified by these experiments:

(a) The bending moments on pulley arms are not evenly distributed by the rim, but are greatest next the tight side of belt.

(b) There are bending moments at both ends of arm, that at the hub being much the greater, the ratio depending on the relative stiffness of rim and arms.

The following rules may be adopted for designing the arms of cast iron pulleys and gears:

1. Multiply the net turning pressure, whether caused by belt or tooth, by a suitable factor of safety and by the length of the arm in inches. Divide this product by *one-half the number of arms* and use the quotient for a bending moment. Design the hub end of arm to resist this moment.

2. Make the rim ends of arms one-half as strong as the hub ends.

64. Safe Speed for Wheels The centrifugal force developed in a rapidly revolving pulley or gear produces a certain tension on the rim, and also a bending of the rim between the arms. We will first investigate the case of a pulley having a rim of uniform cross section.

It is safe to assume that the rim should be capable of bearing its own centrifugal tension without assistance from the arms.

Let D = mean diameter of pulley rim.
t = thickness of rim.
b = breadth of rim.
w = weight of material per cu. in.
= .26 lbs. for cast iron.
= .28 lbs. for wrought iron or steel.
n = number of arms.
N = number revs. per min.
v = velocity of rim in ft. per sec.

First let us consider the centrifugal tension alone. The centrifugal pressure per square inch of concave surface is

$$p = \frac{Wv^2}{gr} \quad \text{.....................} (a)$$

where W is the weight of rim per square inch of concave surface $= wt$, and r = radius in feet $= \frac{D}{24}$

The centrifugal tension produced in the rim by this force is by formula (13)

$$S = \frac{pD}{2t}$$

Substituting the values of p, W and r and reducing:

$$S = \frac{12wv^2}{g} \quad \text{..............} (84)$$

and

$$v = \sqrt{\frac{gS}{12w}} \quad \text{.................} (85)$$

For an average value of $w=.27$, (86) reduces to

$$S = \frac{v^2}{10} \text{ nearly.}$$

a convenient form to remember.

If we assume S as the ultimate tensile strength, 16500 lbs. for cast iron in large castings and 60000 lbs. for soft steel, then the bursting speed of rim is;

for a cast iron wheel

$$v=406 \text{ ft. per sec} \ldots\ldots\ldots\ldots (86)$$

and for steel rim $v=775$ ft. per sec .... (87)

and these values may be used in roughly calculating the safe speed of pulleys.

It has been shown by Mr. James B. Stanwood, in a paper read before the American Society of Mechanical Engineers,* that each section of the rim between the arms is moreover in the condition of a beam fixed at the ends and uniformly loaded.

This condition will produce an additional tension on the outside of rim. The formula for such a beam when of rectangular cross-section is

$$\frac{Wl}{12} = \frac{Sbd^2}{6} \quad \ldots\ldots\ldots\ldots\ldots\ldots (b)$$

W in this case is the centrifugal force of the fraction of rim included between two arms.

The weight of this fraction is $\frac{\pi Dbtw}{n}$ and its centrifugal force $W = \frac{\pi Dbtw}{n} \times \frac{24v^2}{gD}$ or $W = \frac{24\pi btwv^2}{gn}$

Also $$l = \frac{\pi D}{n} \text{ and } d=t$$

* See Trans. A. S. M. E. Vol. XIV.

Substituting these values in (b) and solving for S :

$$S = 3.678 \frac{Dwv^2}{tn^2} \qquad \text{(c)}$$

If w is given an average value of .27 then

$$S = \frac{Dv^2}{tn^2} \text{ nearly} \qquad \text{(d)}$$

and the total value of the tensile stress on outer surface of rim is

$$S' = \frac{Dv^2}{tn^2} + \frac{v^2}{10} \text{ nearly} \qquad (88)$$

Solving for v :

$$v = \sqrt{\frac{S^1}{\frac{D}{tn^2} + \frac{1}{10}}} \qquad (89)$$

In a pulley with a thin rim and small number of arms, the stress due to this bending is seen to be considerable.

It must however be remembered that the stretching of the arms due to their own centrifugal force and that of the rim will to some extent diminish this bending. Mr. Stanwood recommends a deduction of one-half from the value of S in (d) on this account.

Prof. Gaetano Lanza has published quite an elaborate mathematical discussion of this subject. (See Vol. XVI. Trans. Am. Soc. Mech. Engineers.) He shows that in ordinary cases the stretch of the arms will relieve more than one half of the stress due to bending, perhaps three-quarters.

65 Experiments on Fly Wheels. In order to determine experimentally the centrifugal tension and bending in rapidly revolving rims, a large number of small fly wheels have been tested to destruction at the Case School laboratories. In all ten wheels, fifteen inches in dameter and twenty-three wheels two feet in diameter have been so tested. An account of some of these experiments may be found in Trans. Am. Soc. Mech. Eng. Vol. XX. The wheels were all of cast iron and modeled after actual fly wheels. Some had solid rims, some jointed rims and some steel spokes.

To give to the wheels the speed necessary for destruction, use was made of a Dow steam turbine capable of being run at any speed up to 10000 revolutions per minute. The turbine shaft was connected to the shaft carrying the fly wheels by a brass sleeve coupling loosely pinned to the shafts at each end in such a way as to form a universal joint, and so proportioned as to break or slip without injuring the turbine in case of sudden stoppage of the fly wheel shaft.

One experiment with a shield made of two-inch plank convinced us that safety did not lie in that direction, and in succeeding experiments with the fifteen inch wheels a bomb-proof constructed of 6x12 inch white oak was used. The first experiment with a twenty-four inch wheel showed even this to be a flimsy contrivance. In subsequent experiments a shield made of 12x12 inch oak was used. Even this shield was split repeatedly and had to be re-enforced by bolts.

A cast steel ring about four inches thick lined with wooden blocks and covered wiih three inch oak planking was finally adopted.

The wheels were usually demolished by the explosion. No crashing or rending noise was heard, only one quick, sharp report, like a musket shot.

The following tables give a summary of a number of the experiments.

TABLE XVIII. — FIFTEEN INCH WHEELS.

No.	Bursting Speed.		Centrifugal Tension $= \frac{v^2}{10}$	Remarks.
	Revs. per Minute.	Feet per Second = v.		
1	6,525	430	18,500	Six arms.
2	6,525	430	18,500	Six arms.
3	6,035	395	15,600	Thin rim.
4	5,872	380	14,400	Thin rim.
5	2,925	192	3,700	Joint in rim.
6	5,600*	368	13,600	Three arms.
7	6,198	406	16,500	Three arms.
8	5,709	368	13,600	Three arms.
9	5,709	365	13,300	Thin rim.
10	5,709	361	13,000	Thin rim.

* Doubtful.

TABLE XIX.— TWENTY-FOUR INCH WHEELS.

No.	Shape and Size of Rim.					Weight of Wheel, Pounds.
	Diameter Inches	Breadth Inches	Depth Inches	Area Sq. Inches	Style of Joint.	
11	24	$2\frac{1}{8}$	1.5	3.18	Solid rim.	75.25
12	24	$4\frac{1}{16}$	.75	3.85	Internal flanges, bolted	93.
13	24	4	.75	3.85	" " "	91.75
14	24	4	.75	3.85	" " "	95.
15	24	$4\frac{1}{16}$	.75	3.85	" " "	94.75
16	24	1.2	2.1	2.45	Three lugs and links	65.1
17	24	1.2	2.1	2.45	Two lugs and links.	65.

TABLE XX. — FLANGES AND BOLTS.

No.	Flanges. Thickness, Inches.	Flanges. Effective Breadth, Inches.	Flanges. Effective Area, Inches.	Bolts. No. to each Joint.	Bolts. Diameter Inches.	Bolts. Total Tensile Strength, Pounds.
12	$\frac{11}{16}$	2.8	1.92	4	$\frac{5}{16}$	16,000
13	$\frac{11}{16}$	2.75	1.89	4	$\frac{5}{16}$	16,000
14	$\frac{15}{16}$	2.75	2.58	4	$\frac{5}{16}$	16,000
15	$\frac{15}{16}$	2.5	2.34	4	$\frac{3}{8}$	20,000

BY TESTING MACHINE.

Tensile strength of cast iron = 19,600 pounds per square in.
Transverse strength of cast iron = 46,600 pounds per square in.
Tensile strength of $\frac{5}{16}$ bolts = 4,000 pounds.
Tensile strength of $\frac{3}{8}$ bolts = 5,000 pounds.

TABLE XXI. — FAILURE OF FLANGED JOINTS.

No.	Area of Rim, Square Inches.	Effect. Area flanges. Sq. Ins.	Total Strength Bolts, Pounds.	Bursting Speed. Rev. per Min.	Bursting Speed. Ft. per Sec. = v.	Cent. Tension. Per Sq. In. $=\frac{v^2}{10}$	Cent. Tension. Total Lbs.	Remarks.
11	3.18	. .	. . .	3,672	385	14,800	47,000	Solid rim.
12	3.85	1.92	16,000	. . .	. .	. . .	. . .	Flange broke.
13	3.85	1.89	16,000	1,760	184	3,400	13,100	Flange broke.
14	3.85	2.58	16,000	1,875	196	3,850	14,800	Bolts broke.
15	3.85	2.34	20,000	1,810	190	3,610	13,900	Flange broke.

TABLE XXII.—LINKED JOINTS.

No.	Lugs.			Links.				Rim.	
	Breadth, Inches.	Length Inches.	Area Sq. In.	Number Used.	Effect Breadth. Inches.	Thickness Inches.	Effective Area, Sq. Ins.	Max. Area, Sq. Ins.	Net Area, Sq. Ins.
16	.45	1.0	.45	3	.57	.327	.186	2.45	1.98
17	.44	.98	.43	2	.54	.380	.205	2.45	1.98

BY TESTING MACHINE.

Tensile strength of cast iron = 19,600.
Transverse strength of cast iron = 40,400.
Av. tensile strength of each link = 10,180.

TABLE XXIII.—FAILURE OF LINKED JOINTS.

No.	Strength of Links, Pounds.	Strength of Rim, Pounds.	Bursting Speed.		Cent. Tension.		Remarks.
			Rev. per Min.	Ft. per Sec. $=v$	Per Sq. In. $=\frac{v^2}{10}$	Total.	
16	30,540	38,800	3,060	320	10,240	25,100	Rim broke.
17	20,360	38,800	2,750	290	8,410	20,600	Lugs and Rim broke.

The flanged joints mentioned had the internal flanges and bolts common in large belt wheel rims while the linked joints were such as are commonly used in fly wheels not used for belts.

Subsequent experiments have given approximately the same results as those just detailed. The highest velocity yet attained has been 424 feet per second; this is in a solid cast iron rim with numerous steel spokes. The average bursting velocity for solid cast rims with cast spokes is 400 feet per second.

Wheels with jointed rims burst at speeds varying from 190 to 250 feet per second, according to the style of joint and its location. The following general conclusions seem justified by these tests.

1. Fly-wheels with solid rims, of the proportions usual among engine builders and having the usual number of arms, have a sufficient factor of safety at a rim speed of 100 feet per second if the iron is of good quality and there are no serious cooling strains.

In such wheels the bending due to centrifugal force is slight, and may safely be disregarded.

2. Rimjoints midway between the arms are a serious defect and reduce the factor of safety very materially. Such joints are as serious mistakes in design as would be a joint in the middle of a girder under a heavy load.

3. Joints made in the ordinary manner, with internal flanges and bolts, are probably the worst that could be devised for this purpose. Under the most favorable circumstances they have only about one-fourth the strength of the solid rim and are particularly weak against bending.

In several joints of this character, on large fly-wheels, calculation has shown a strength less than one-fifth that of the rim.

4. The type of joint known as the link or prisoner joint is probably the best that could be devised for

narrow rimmed wheels not intended to carry belts, and possesses when properly designed a strength about two-thirds that of the solid rim.

66. Rims of Cast Iron Gears. A toothed wheel will burst at a less speed than a pulley because the teeth increase the weight and therefore the centrifugal force without adding to the strength.

The centrifugal force and therefore the stresses due to the force will be increased nearly in the ratio that the weight of rim and teeth is greater than the weight of rim alone.

This ratio in ordinary gearing varies from 1.5 to 1.7. We will assume 1.6 as an average value. Neglecting bending we now have from equation (86)

$$S = 1.6 \times \frac{12wv^2}{g} = \frac{19.2wv^2}{g} \qquad (90)$$

and

$$v = \sqrt{\frac{gS}{19.2w}}$$

$$= 326.2 \text{ ft. per second} \qquad (91)$$

Including bending

$$S' = 1.6v^2 \left\{ \frac{D}{tn^2} + \frac{1}{10} \right\} \qquad (92)$$

As the transverse strength of cast iron by experiment is about double the tensile strength, a larger value of S may be allowed in formulas (88), (89) and (92).

In built up wheels it is better to have the joints come near the arms to prevent the tendency of the bending to open the joints, and the fastenings should have the same tensile strength as the rim of the wheel.

EXAMPLES.

1. Design eight arms of elliptic section for a gear 48 ins. pitch diameter to transmit a pressure on tooth of 800 lbs.

2. Determine bursting speed of the gear in previous example in revolutions per minute if the thickness of rim is .75 inch.

(1) Considering centrifugal tension alone.

(2) Including bending of rim due to centrifugal force, assuming that one-half the stress due to bending is relieved by the stretching of the arms.

3. Design a link joint for the rim of a fly-wheel, the rim being 8 ins. wide, 12 ins. deep and 18 ft. mean diameter, the links to have a tensile strength of 65000 lbs. per sq. in. Determine the relative strength of joint and the probable bursting speed.

4. Discuss the proportions of one of the following wheels in the laboratory and criticise dimensions.

(a) Fly-wheel, Allis engine.
(b) Fly-wheel, Fairbanks gas engine.
(c) Large Medart pulleys, Electrical laboratory.
(d) Belt-wheel, Allis engine.

Chapter II.

TRANSMISSION BY BELTS AND ROPES.

67. Friction of Belting. The transmitting power of a belt is due to its friction on the pulley, and this friction is equal to the difference between the tensions of the driving and slack sides of the belt.

Let w = width of belt.
T_1 = tension of driving side.
T_2 = tension of slack side.
R = friction of belt $= T_1 - T_2$
f = co-efficient of friction between belt and pulley.
θ = arc of contact in circular measure.

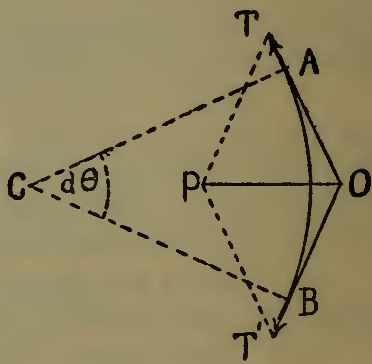


Fig. 40.

The tension T at any part of the arc of contact is intermediate between T_1 and T_2.

Let AB Fig. 40, be an indefinitely short element of the arc of contact, so that the tensions at A and B differ only by the amount dT.

dT will then equal the friction on AB which we may call dR.

Draw the intersecting tangents OT and OT′ to represent the tensions and find their radial resultant OP. Then will OP represent the normal pressure on the arc AB which we will call P.

$$\angle OTP = \angle ACB = d\theta$$
$$\therefore P = Td\theta$$

The friction on AB is

$$f\,P = fTd\theta$$

or

$$dT = dR = fTd\theta$$

and

$$fd\theta = \frac{dT}{T}$$

Integrating for the whole arc θ:

$$f\theta = \int_{T_2}^{T_1} \frac{dT}{T} = \log_e \frac{T_1}{T_2}$$

$$\frac{T_1}{T_2} = e^{f\theta}$$

$$T_2 = \frac{T_1}{e^{f\theta}} = T_1 e^{-f\theta}$$

$$R = T_1 - T_2 = T_1(1 - e^{-f\theta}) \quad \dots\dots\dots\dots\dots\dots (93)$$

The average value of f for leather belts on iron pulleys as determined by experiment is $f = 0.27$

If we denote the expression $(1 - e^{-f\theta})$ by C, then for different arcs of contact C has the following values:

Arc of Contact.	90°	110°	130°	150°	180°	210°	240°
C	.345	.404	.458	.506	.571	.627	.676

The friction or force transmitted by a belt per inch of width is then

$$R = CT_1 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (94)$$

and T_1 must not exceed the safe working tensile strength of the material.

A handy rule for calculating belts assumes C=.5 which means that the force which a belt will transmit under ordinary conditions is one-half its tensile strength.

68 Strength of Belting. The strength of belting varies widely and only average values can be given. According to experiments made by the author good oak tanned belting has a breaking strength per inch of width as follows:

	Single.	Double.
Solid leather	900 lbs.	1400 lbs.
Where riveted	600 lbs.	1200 lbs.
Where laced	350 lbs.	....

Canvas belting has approximately the same strength as leather. Tests of rubber coated canvas belts 4-ply, 8 inches wide, show a tensile strength of from 840 lbs to 930 lbs. per inch of width.

69. Taylor's Experiments. The experiments of Mr. F. W. Taylor, as reported by him in Trans. Am. Soc. Mech. Eng. Vol. XV. afford the most valuable data now available on the performance of belts in actual service.

These experiments were carried on during a period of nine years at the Midvale Steel Works. Mr. Taylor's conclusions may be epitomized as follows :

1. Narrow double belts are more economical than single ones of a greater width.

2. All joints should be spliced and cemented.

3. The most economical belt speed is from 4000 to 4500 ft. per min.

4. The working tension of a double belt should not exceed 35 lbs. per inch of width, but the belt may be first tightened to about double this.

5. Belts should be cleaned and greased every six months.

6. The best length is from 20 to 25 feet between centers.

70. Rules for Width of Belts It will be noticed that Mr. Taylor recommends a working tension only $\frac{1}{30}$ to $\frac{1}{10}$ the breaking strength of the belt. He justifies this by saying that belts so designed gave much less trouble from stoppage and repairs and were consequently more economical than those designed by the ordinary rules.

In the following formulas 50 lbs. per inch of width is allowed for double belts and 30 lbs. for single belts. These are suitable values for belts which are not running continuously. The formulas may be easily changed for other thicknesses and for other values of CT_1.

Let HP = horse power transmitted.
D = diameter of driving pulley in inches.
N = no. revs. per min. of pulley.

The moment of force transmitted by belt is

$$\frac{RD}{2} = \frac{CT_1wD}{2} = T$$

and

$$HP = \frac{TN}{63025} = \frac{CT_1wDN}{126050} \quad \ldots\ldots\ldots\ldots (95)$$

Substituting the values assumed for CT_1 and solving for w :

$$\text{Single belts } w = 4200\frac{HP}{DN} \quad \ldots\ldots\ldots\ldots (96)$$

$$\text{Double belts } w = 2500\frac{HP}{DN} \quad \ldots\ldots\ldots\ldots (97)$$

The most convenient rules for belting are those which give the horse-power of a belt in terms of the surface passing a fixed point per minute.

In formula (95)

$$HP = \frac{CT_1wDN}{126\,050}$$

we will substitute the following :

$$W = \text{width of belt in feet} = \frac{w}{12}$$

$$V = \text{velocity in ft. per min.} = \frac{\pi DN}{12}$$

or

$$HP = \frac{144CT_1WV}{126050\pi}$$

Substituting values of C and T_1 as before and solving for WV=square feet per minute we have approximately:

$$\text{Single belts } WV = 90\ HP \text{ ....(98)}$$

$$\text{Double belts } WV = 55\ HP \text{ ....(99)}$$

71. **Speed of Belting.** As in the case of pulley rims, so in that of belts a certain amount of tension is caused by the centrifugal force of the belt as it passes around the pulley.

From equation (84)

$$S = \frac{12wv^2}{g}$$

where
v=velocity in ft. per sec.
w=weight of material per cu. in.
S=tensile stress per sq. in.

To make this formula more convenient for use we will make the following changes in the constants:

Let V=velocity of belt in ft. per minute=60v.
w=weight of ordinary belting.
=.032 per cu. in.
S_1=tensile stress per inch width, caused by centrifugal force.
=about $\frac{3}{16}$ S for single belts.

Then

$$v = \frac{V}{60}$$

$$S = \frac{16S_1}{3}$$

Substituting these values in (86) and solving for S_1

$$S_1 = \frac{V^2}{1\,610\,000} \quad \text{.......... .....} \quad (100)$$

The speed usually given as a safe limit for ordinary belts is 3000 ft. per min., but belts are sometimes run at a speed exceeding 6000 ft. per min.

Substituting different values of V in the formula we have :

$V=3000$	$S_1= 5.59$ lbs.
$V=4000$	$S_1= 9.94$ lbs.
$V=5000$	$S_1=15.53$ lbs.
$V=6000$	$S_1=22.36$ lbs.

The values of S_1 for double belts will be nearly twice those given above. At a speed of 5000 ft. per minute the maximum tension per inch of width on a single belt designed by formula (96), if we call $C = .5$, will be :

$$(30 \times 2) + 15. = 75 \text{ lbs.}$$

giving a factor of safety of eight or ten at the splices.

In a similar manner we find the maximum tension per inch of width of a double belt to be :

$$(50 \times 2) + 30 = 130 \text{ lbs.}$$

and the margin of safety about the same as in single belting.

72. Manila Rope Transmission. Ropes are sometimes used instead of flat belts for transmitting power short distances. They possess the following advantages: they are cheaper than belts in first cost; they are flexible in every direction and can be carried around corners readily. They have however the disadvantage of being less efficient in transmission than leather belts and less durable; they are also somewhat difficult to splice or repair.

There are two systems of rope driving in common use: the English and the American. In the former there are as many separate ropes as there are grooves in one pulley, each rope being an endless loop always running in one groove.

In the American system one continuous rope is used passing back and forth from one groove to another and finally returning to the starting point.

The advantage of the English system consists in the fact that one of the ropes may fail without causing a breakdown of the entire drive, there usually being two or three ropes in excess of the number actually necessary. On the other hand the American system has the advantage of a uniform regulation of the tension on all the plies of rope. The guide pulley, which guides the last slack turn of rope back to the starting point, is usually also a tension pulley and can be weighted to secure any desired tension. The English method is most used for heavy drives from engines to head shafts; the American for lighter work in distributing power to the different rooms of a factory. The grooves in the pulleys for manila or cotton ropes usually have their sides inclined at an angle of about 45°, thus wedging the rope in the groove.

The Walker groove has curved sides as shown in Fig. 41, the curvature increasing towards the bottom. As the rope wears and stretches it becomes smaller and sinks deeper in the groove; the sides of the groove being more oblique near the bottom, the older rope is not pinched so hard as the newer and this tends to throw more of the work on the latter.

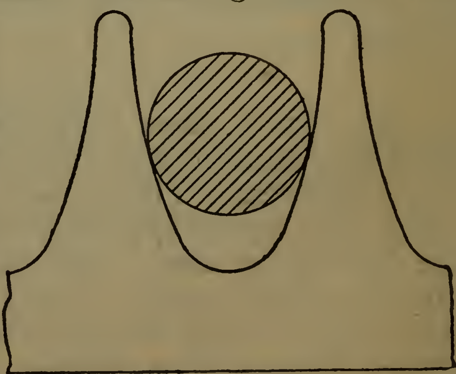


Fig. 41.

73. Strength of Manila Ropes. The formulas for transmission by ropes are similar to those for belts the values for S and φ being changed. The ultimate tensile strength of manila and hemp rope is about 10000 lbs. per sq. in.

To insure durability and efficiency it has been found best in practice to use a large factor of safety. Prof. Forrest R. Jones in his book on Machine Design recommends a maximum tension of 200 d^2 pounds where d is the diameter of rope in inches. This corresponds to a tensile stress of 255 lbs. per sq. in. or a factor of safety of about 40.

The value of f for manila on metal is about 0.12, but as the normal pressure between the two surfaces is increased by the wedge action of the rope in the groove we shall have the apparent value of f:

$$f^1 = f \div \sin\frac{\alpha}{2} \text{ where}$$

$$\alpha = \text{angle of groove,}$$

For $\alpha = 45^\circ$ to 30°

f^1 varies from 0.3 to 0.5 and these values should be used in formula (93).

$(1 - e^{-f\theta})$ in this formula for an arc of contact of 150°, becomes either .54 or .73 according as f^1 is taken 0.3 or 0.5.

If T_1 is assumed as 250 lbs. per sq. in., the force R transmitted by the rope varies from 135 lbs to 185 lbs. per sq. in. area of rope section.

The following table gives the horse-power of manila ropes based on a maximum tension of 255 lbs. per sq. in.

TABLE XXIV.

Table of the horse-power of transmission rope, reprinted from the transactions of the American Society of Mechanical Engineers, Vol. 12, page 230, Article on "Rope Driving" by C. W. Hunt.

The working strain is 800 lbs. for a 2 inch diameter rope and is the same at all speeds, due allowance having been made for loss by centrifugal force.

Diameter Rope, Inches.	SPEED OF THE ROPE IN FEET PER MINUTE.										Smallest Diam. Pulleys, Ins.
	1500	2000	2500	3000	3500	4000	4500	5000	6000	7000	
¾	3.3	4.3	5.2	5.8	6.7	7.2	7.7	7.7	7.1	4.9	30
⅞	4.5	5.9	7.0	8.2	9.1	9.8	10.8	10.8	9.3	6.9	36
1.	5.8	7.7	9.2	10.7	11.9	12.8	13.6	13.7	12.5	8.8	42
1¼	9.2	12.1	14.3	16.8	18.6	20.0	21.2	21.4	19.5	13.8	54
1½	13.1	17.4	20.7	23.1	26.8	28.8	30.6	30.8	28.2	19.8	60
1¾	18.0	23.7	28.2	32.8	36.4	39.2	41.5	41.8	37.4	27.6	72
2	23.1	30.8	36.8	42.8	47.6	51.2	54.4	54.8	50.0	35.2	84

74. Wire Rope Transmission. Wire Ropes have been used to transmit power where the distances were too great for belting or hemp rope transmission. The increased use of electrical transmission is gradually crowding out this latter form of rope driving.

For comparatively short distances of from 100 to 500 yards wire rope still offers a cheap and simple means of carrying power.

The pulleys or wheels are entirely different from those used with manila ropes.

Fig. 42 shows a section of the rim of such a pulley. The rope does not touch the sides of the groove but rests on a shallow depression in a wooden, leather or rubber filling at the bottom. The high side flanges prevent the rope from leaving the pulley when swaying on account of the high speed.

The pulleys must be large, usually about 100 times the diameter of rope used, and run at comparatively high speeds. The ropes should not be less than 200 feet long unless some form of tightening pulley is used. — Table XXV is taken from Roebling.

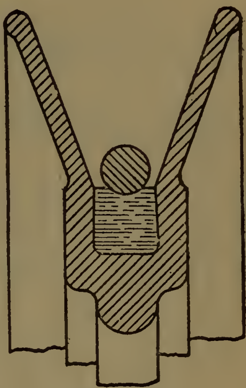


Fig. 42.

Long ropes should be supported by idle pulleys every 400 feet.

EXAMPLES.

1. Design a main driving belt to transmit 150 HP from a belt wheel 18 ft. in diameter and making 80 revs. per min. The belt to be double leather without rivets.

2. Investigate driving belt on Allis engine and calculate the horse-power it is capable of transmitting economically.

3. Calculate the total maximum tension per inch of width due to load and to centrifugal force of the driving belt on the generator used for lighting, assuming the maximum load to be 50 HP.

4. Design a manila rope drive, English system, to transmit 500 HP, the wheel on the engine being 20 feet in diameter and making 60 revs. per min. Use

TABLE XXV.

TRANSMISSION OF POWER BY WIRE ROPES.

Showing necessary size and speed of wheels and rope to obtain any desired amount of power.

Diameter of Wheel in ft.	Number of Revolutions.	Diameter of Rope.	Horse-Power.	Diameter of Wheel in ft.	Number of Revolutions.	Diameter of Rope.	Horse-Power.
4	80	3/8	3.3	10	80	11-16	58.4
	100	3/8	4.1		100	11-16	73.
	120	3/8	5.		120	11-16	87.6
	140	3/8	5.8		140	11-16	102.2
5	80	7-16	6.9	11	80	11-16	75.5
	100	7-16	8.6		100	11-16	94.4
	120	7-16	10.3		120	11-16	113.3
	140	7-16	12.1		140	11-16	132.1
6	80	1/2	10.7	12	80	3/4	99.3
	100	1/2	13.4		100	3/4	124.1
	120	1/2	16.1		120	3/4	148.9
	140	1/2	18.7		140	3/4	173.7
7	80	9-16	16.9	13	80	3/4	122.6
	100	9-16	21.1		100	3/4	153.2
	120	9-16	25.3		120	3/4	183.9
8	80	5/8	22.	14	80	7/8	148.
	100	5/8	27.5		100	7/8	185.
	120	5/8	33.0		120	7/8	222.
9	80	5/8	41.5	15	80	7/8	217.
	100	5/8	51.9		100	7/8	259.
	120	5/8	62.2		120	7/8	300.

Hunt's table and then check by calculating the centrifugal tension and the total maximum tension on each rope.

5. Design a wire rope transmission to carry 120 H P a distance of one-quarter mile using two ropes. Determine working and maximum tension on rope, length of rope, diameter and speed of pulleys and number of supporting pulleys.

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